[AMERICAN JOURNAL OF SCIENCE, VOL. 274, DECEMBER, 1974, P. 1199-1261]

# THEORETICAL PREDICTION OF THE THERMODYNAMIC BEHAVIOR OF AQUEOUS ELECTROLYTES AT HIGH PRESSURES AND TEMPERATURES: II. DEBYE-HÜCKEL PARAMETERS FOR ACTIVITY COEFFICIENTS AND RELATIVE PARTIAL MOLAL PROPERTIES

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ABSTRACT. Electrostatic parameters in the Debye-Hückel equations for activity coefficients and the relative partial molal enthalpies, internal energies, isochoric and isobaric heat capacities, volumes, compressibilities, and expansibilities of aqueous electrolytes have been computed for pressures and temperatures from 1 bar to 5 kb and 0° to 600°C. The calculations are based on the equation of state for $H_2O$ derived by Keenan and others (1969) for pressures $\leq$ a kilobar together with power functions of temperature and density (or pressure) obtained by regression analysis (Helgeson and Kirkham, 1974a) of dielectric constant data (Oshry, ms; Owen and others, 1961; Heger, ms) and finite difference coefficients of isobaric thermal expansion and isothermal compressibility generated (Helgeson and Kirkham, 1974a) from densities (Burnham, Holloway, and Davis, 1969) of $H_2O$ at pressures $>$ a kilobar. The Debye-Hückel parameters are sensitive quasi-asymptotic functions of pressure and temperature, which may be monotonic or exhibit extrema, depending on the property. Changes in the dielectric constant and the Debye-Hückel parameters with increasing temperature and/or decreasing pressure favor increasing mineral solubilities and extension of the applicability of Debye-Hückel theory to electrolytes with progressively higher concentrations. In the supercritical region, the absolute values of the limiting law parameters for the relative partial molal properties of electrolytes are of the order of 5 to $>$ 1000 times larger than they are at 25°C and 1 bar.

## INTRODUCTION

Calculation of the partial molal properties of aqueous electrolyte solutions as well as controlled extrapolation to infinite dilution of apparent molal properties computed from experimental data requires values for the electrostatic parameters in the Debye-Hückel theory (Debye and Hückel, 1923) of electrolytes. The purpose of this communication is to present a comprehensive summary of these parameters for temperatures and pressures from 0° to 600°C and 1 bar to 5 kb. The results of the calculations are presented in diagrams as well as tables to facilitate analysis of the nature and extent to which the parameters change and the role they play in high pressure-temperature solution chemistry.

Despite the fact that aqueous electrolytes become associated at high temperatures and pressures, the Debye-Hückel formulation is applicable if the "true" ionic strength of the solution can be computed. Ionic strength calculations can be carried out with the aid of high pressure-temperature conductance measurements such as those reported by Hensel and Franck (1964) and Quist and Marshall (1966; 1968a, b, c, and d; 1969), which make it possible to compute the degrees of formation of complexes in electrolyte solutions as a function of pressure and temperature. Although considerable discussion in the literature has been devoted to the applicability of the dielectric constant of pure $H_2O$ to represent

the electrostatic environment of solute species in dilute aqueous solutions, the success of continuum theory (Sen and Cobble, 1974) and the solvated ion model adopted by Robinson and Stokes (1959) afford reasonable justification for doing so.

With few exceptions, previously reported values of Debye-Hückel parameters (for example, Robinson and Stokes, 1959; Harned and Owen, 1958; Lewis and Randall, 1961; Marshall and Slusher, 1968; Bromley, 1968; Bromley and others, 1970; Gardner, Mitchell, and Cobble, 1969; Redlich and Meyer, 1964; Hamer, 1968; Hamer and Wu, 1972; Lietzke and Stoughton, 1962; Helgeson, 1967b) are limited to pressures and temperatures along the saturation curve for water. Many of these are inconsistent with one another owing to use of different experimental data and/or power functions to represent the dielectric constant, density, and their partial derivatives as functions of temperature and pressure. In contrast, the calculations reported below are based on an internally consistent set of values for the thermodynamic/electrostatic properties of $H_2O$.

## UNITS, CONVENTIONS, AND UNCERTAINTIES

Except where confusion might arise (as, for example, in the use of E to represent internal energy as well as expansibility), the definitions, symbols, and notation employed below conform (or are analogous) to those used by Lewis and Randall (1961). In certain cases arbitrary sign conventions (given below) were adopted for convenience. Energy is expressed in thermochemical calories (4.184 cal = 1 joule) or kcal $mole^{-1}$ and heat capacity in cal $mole^{-1}$ $(°K)^{-1}$. All temperatures are on the thermodynamic (celsius) scale, expressed in degrees kelvin (°K) or degrees centigrade (°C). The values of the gas constant employed in the calculations are 1.98719 cal $mole^{-1}$ $(°K)^{-1}$ or 83.14241 $cm^3$ bar $mole^{-1}$ $(°K)^{-1}$.

Although volumetric concentration units are specified in Debye-Hückel theory, molality units of concentration are generally more convenient for calculations in which pressure and temperature are variables. Despite recent arguments to the contrary (Quist and Marshall, 1968a), no concentration scale is intrinsically more correct than another. In contrast to molarity, molality is independent of temperature and pressure, which facilitates calculation and comparison of activity coefficients and the relative partial molal properties of aqueous electrolytes at different concentrations, pressures, and temperatures. For this reason, all properties given below are expressed in molality units, which can be converted to the molarity scale by dividing the values of the parameters by the square root of the density of $H_2O$. The standard state adopted in this study is the conventional one of unit activity of the solute in a hypothetical one (mean) molal solution. At infinite dilution, activity coefficients are taken to be unity at all pressures and temperatures. The labels *sat* and *saturation* in the tables and diagrams refer to steam-saturated liquid $H_2O$.

The density and dielectric constant of $H_2O$ together with their partial derivatives have been computed as functions of pressure and temperature from 0.001 to 5 kb and 25° to 600°C (Helgeson and Kirkham, 1974a). The results of these calculations were used in the present study to calculate the Debye-Hückel parameters shown in the following tables and diagrams.

Because the temperature and pressure dependence of many of the Debye-Hückel parameters is controlled by the relative size of the partial derivatives of ln $\varepsilon$ and ln V with respect to temperature or pressure, these two variables are plotted against each other in figure 1, where it can be seen that ln $\varepsilon$ decreases as ln V increases, either at constant pressure or constant temperature. Note that in the latter case the two variables are almost linearly related with a slope close to unity. This relation not only has an important effect on the behavior of the Debye-Hückel parameters for relative partial molal volumes and compressibilities as functions of pressure and temperature (discussed below), but it also minimizes the effect of absolute uncertainties in calculated values of the partial derivatives of ln V and ln $\varepsilon$ on derived values of Debye-Hückel parameters. The latter variables are affected to a much greater extent by the *relative* uncertainties in the computed values of the derivatives, which are smaller than their absolute counterparts.

Uncertainties in the calculated Debye-Hückel parameters reported in the following pages are difficult to assess, but they are of the order of 1 to 5 percent or less, except for $450° \lesssim t \leq 550°C$ and $1000 < P \lesssim 1500$

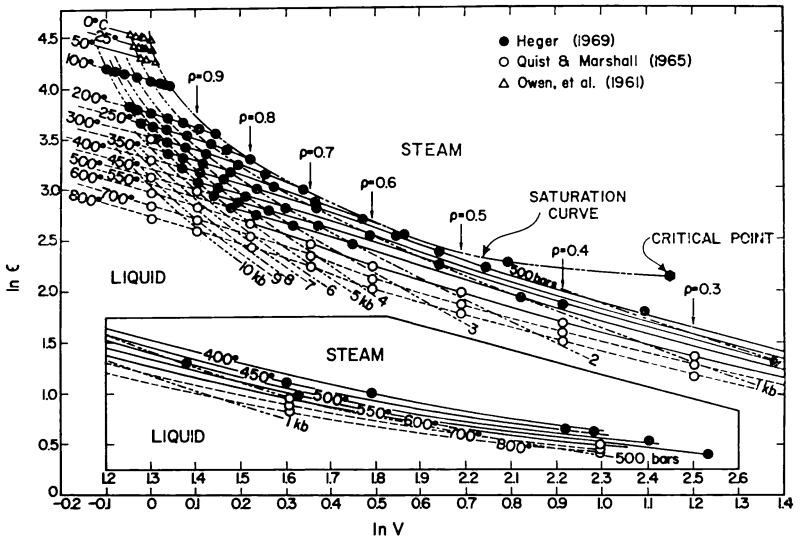


Fig. 1. Logarithm of the dielectric constant ($\varepsilon$) of $H_2O$ as a function of the logarithm of the specific volume (V) of $H_2O$ at constant temperature (solid curves) and constant pressure (dashed curves) calculated from regression equations (Helgeson and Kirkham, 1974a). The symbols represent values taken from the literature.

TABLE 1

$A_\gamma$ in $kg^{1/2}mole^{-1/2}$ computed from equation (2)—see figures 2, 3, and 6

| t (°C) | PRESSURE, KB | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | SAT | 0.5 | 1 | 1.5 | 2 | 3 | 4 | 5 |
| 25 | 0.5092 | 0.4980 | 0.4882 | (0.4807) | (0.4730) | (0.4606) | (0.4505) | (0.4417) |
| 50 | 0.5336 | 0.5208 | 0.5096 | (0.5013) | (0.4925) | (0.4784) | (0.4670) | (0.4570) |
| 75 | 0.5639 | 0.5486 | 0.5353 | (0.5251) | (0.5149) | (0.4987) | (0.4860) | (0.4748) |
| 100 | 0.5998 | 0.5809 | 0.5649 | 0.5522 | 0.5402 | 0.5214 | 0.5070 | 0.4947 |
| 125 | 0.6416 | 0.6177 | 0.5983 | 0.5828 | 0.5686 | 0.5463 | 0.5299 | 0.5161 |
| 150 | 0.6898 | 0.6592 | 0.6352 | 0.6168 | 0.5998 | 0.5732 | 0.5541 | 0.5387 |
| 175 | 0.7454 | 0.7057 | 0.6756 | 0.6538 | 0.6334 | 0.6016 | 0.5795 | 0.5621 |
| 200 | 0.8099 | 0.7576 | 0.7196 | 0.6934 | 0.6692 | 0.6315 | 0.6057 | 0.5863 |
| 225 | 0.8860 | 0.8159 | 0.7673 | 0.7353 | 0.7068 | 0.6625 | 0.6327 | 0.6111 |
| 250 | 0.9785 | 0.8822 | 0.8192 | 0.7795 | 0.7461 | 0.6946 | 0.6605 | 0.6366 |
| 275 | 1.0960 | 0.9595 | 0.8762 | 0.8263 | 0.7873 | 0.7280 | 0.6894 | 0.6629 |
| 300 | 1.2555 | 1.0529 | 0.9398 | 0.8766 | 0.8308 | 0.7630 | 0.7195 | 0.6905 |
| 325 | 1.4943 | 1.1705 | 1.0126 | 0.9317 | 0.8774 | 0.8001 | 0.7515 | 0.7198 |
| 350 | 1.9252 | 1.3267 | 1.0981 | 0.9934 | 0.9282 | 0.8399 | 0.7857 | 0.7513 |
| 375 | | 1.5464 | 1.2007 | 1.0639 | 0.9845 | 0.8832 | 0.8228 | 0.7854 |
| 400 | | 1.8789 | 1.3262 | 1.1453 | 1.0476 | 0.9305 | 0.8632 | 0.8225 |
| 425 | | 2.4301 | 1.4811 | 1.2402 | 1.1188 | 0.9823 | 0.9069 | 0.8625 |
| 450 | | 3.3553 | 1.6723 | 1.3504 | 1.1988 | 1.0384 | 0.9536 | 0.9049 |
| 475 | | 4.5059 | 1.9065 | 1.4768 | 1.2873 | 1.0977 | 1.0017 | 0.9480 |
| 500 | | 5.5075 | 2.1872 | 1.6181 | 1.3822 | 1.1575 | 1.0485 | 0.9889 |
| 525 | | | (2.5092) | (1.7685) | (1.4782) | (1.2127) | (1.0890) | (1.0228) |
| 550 | | | (2.8476) | (1.9151) | (1.5648) | (1.2551) | (1.1158) | (1.0428) |
| 575 | | | (3.1486) | (2.0366) | (1.6259) | (1.2735) | (1.1199) | (1.0406) |
| 600 | | | (3.3281) | (2.0960) | (1.6400) | (1.2556) | (1.0919) | (1.0081) |

TABLE 2

$B_\gamma$ in $(kg^{1/2}mole^{-1/2}\ cm^{-1}) \times 10^{-8}$ computed from equation (3)—see figures 4, 5, and 7

| t (°C) | PRESSURE, KB | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | SAT | 0.5 | 1 | 1.5 | 2 | 3 | 4 | 5 |
| 25 | 0.3283 | 0.3282 | 0.3281 | (0.3281) | (0.3280) | (0.3297) | (0.3297) | (0.3280) |
| 50 | 0.3325 | 0.3321 | 0.3317 | (0.3315) | (0.3312) | (0.3308) | (0.3306) | (0.3304) |
| 75 | 0.3371 | 0.3364 | 0.3358 | (0.3353) | (0.3349) | (0.3342 | (0.3337) | 0.3334 |
| 100 | 0.3422 | 0.3411 | 0.3402 | 0.3395 | 0.3388 | 0.3378 | 0.3372 | 0.3366 |
| 125 | 0.3476 | 0.3461 | 0.3448 | 0.3439 | 0.3430 | 0.3416 | 0.3407 | 0.3401 |
| 150 | 0.3533 | 0.3512 | 0.3496 | 0.3483 | 0.3472 | 0.3455 | 0.3443 | 0.3435 |
| 175 | 0.3592 | 0.3565 | 0.3544 | 0.3528 | 0.3514 | 0.3493 | 0.3479 | 0.3469 |
| 200 | 0.3655 | 0.3618 | 0.3592 | 0.3573 | 0.3556 | 0.3530 | 0.3514 | 0.3503 |
| 225 | 0.3721 | 0.3673 | 0.3639 | 0.3616 | 0.3596 | 0.3566 | 0.3547 | 0.3535 |
| 250 | 0.3792 | 0.3729 | 0.3686 | 0.3658 | 0.3635 | 0.3601 | 0.3580 | 0.3567 |
| 275 | 0.3871 | 0.3787 | 0.3733 | 0.3699 | 0.3673 | 0.3634 | 0.3611 | 0.3598 |
| 300 | 0.3965 | 0.3850 | 0.3780 | 0.3739 | 0.3710 | 0.3667 | 0.3643 | 0.3629 |
| 325 | 0.4085 | 0.3921 | 0.3829 | 0.3780 | 0.3747 | 0.3700 | 0.3674 | 0.3661 |
| 350 | 0.4256 | 0.4004 | 0.3882 | 0.3822 | 0.3784 | 0.3734 | 0.3707 | 0.3694 |
| 375 | | 0.4104 | 0.3940 | 0.3867 | 0.3823 | 0.3769 | 0.3741 | 0.3729 |
| 400 | | 0.4230 | 0.4004 | 0.3915 | 0.3865 | 0.3806 | 0.3777 | 0.3766 |
| 425 | | 0.4386 | 0.4076 | 0.3968 | 0.3909 | 0.3845 | 0.3815 | 0.3804 |
| 450 | | 0.4548 | 0.4154 | 0.4024 | 0.3956 | 0.3885 | 0.3853 | 0.3843 |
| 475 | | 0.4625 | 0.4237 | 0.4083 | 0.4004 | 0.3924 | 0.3890 | 0.3880 |
| 500 | | 0.4620 | 0.4321 | 0.4141 | 0.4050 | 0.3960 | 0.3923 | 0.3912 |
| 525 | | | (0.4397) | (0.4193) | (0.4089) | (0.3986) | (0.3944) | (0.3932) |
| 550 | | | (0.4454) | (0.4231) | (0.4113) | (0.3995) | (0.3948) | (0.3934) |
| 575 | | | (0.4472) | (0.4240) | (0.4109) | (0.3978) | (0.3924) | (0.3907) |
| 600 | | | (0.4428) | (0.4203) | (0.4064) | (0.3922) | (0.3863) | (0.3842) |

TABLE 3

$A_H$ in kcal $kg^{1/2}mole^{-3/2}$ computed from equations (2), (5), and (7)—see figures 8 through 10 and 13

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | 0.692 | 0.65 | 0.62 | (0.56) | (0.51) | (0.47) | (0.44) |
| 50 | 1.049 | 0.97 | 0.90 | (0.80) | (0.73) | (0.68) | (0.63) |
| 75 | 1.471 | 1.33 | 1.23 | (1.07) | (0.97) | (0.90) | (0.84) |
| 100 | 1.981 | 1.76 | 1.61 | 1.38 | 1.24 | 1.14 | (1.06) |
| 125 | 2.610 | 2.27 | 2.04 | 1.71 | 1.52 | 1.39 | (1.29) |
| 150 | 3.403 | 2.88 | 2.54 | 2.08 | 1.82 | 1.65 | (1.53) |
| 175 | 4.427 | 3.61 | 3.10 | 2.49 | 2.14 | 1.92 | (1.78) |
| 200 | 5.805 | 4.50 | 3.75 | 2.94 | 2.47 | 2.21 | (2.04) |
| 225 | 7.773 | 5.63 | 4.51 | 3.44 | 2.83 | 2.51 | (2.31) |
| 250 | 10.815 | 7.13 | 5.43 | 4.00 | 3.24 | 2.84 | (2.62) |
| 275 | 15.981 | 9.27 | 6.60 | 4.66 | 3.71 | 3.24 | (2.98) |
| 300 | 25.894 | 12.48 | 8.15 | 5.45 | 4.27 | 3.71 | (3.42) |
| 325 | 49.072 | 17.54 | 10.28 | 6.42 | 4.96 | 4.29 | (3.96) |
| 350 | 135.177 | 25.93 | 13.25 | 7.63 | 5.82 | 5.01 | (4.63) |
| 375 | | | 17.39 | 9.16 | 6.87 | 5.88 | (5.43) |
| 400 | | | 23.07 | 11.05 | 8.12 | 6.90 | (6.35) |
| 425 | | | 30.67 | 13.34 | 9.54 | 8.00 | (7.32) |
| 450 | | | (40.53) | (15.96) | (11.00) | (9.05) | (8.19 |
| 475 | | | (52.73) | (18.67) | (12.22) | (9.77) | (8.68) |
| 500 | | | (66.41) | (20.92) | (12.69) | (9.68) | (8.34) |
| 525 | | | (78.48) | (21.84) | (11.67) | (8.17) | (6.59) |
| 550 | | | (82.22) | (19.20) | (8.02) | (4.25) | (2.56) |
| 575 | | | (67.48) | (11.02) | (0.60) | (-2.76) | (-4.26) |
| 600 | | | (24.13) | (-4.54) | (-11.10) | (-12.99) | (-13.81) |

TABLE 4

$B_H$ in (cal $kg^{1/2}cm^{-1}mole^{-3/2}$) $\times 10^{-9}$ computed from equations (3), (6), and (8)—see figures 11, 12, and 14

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | 12.60 | 11.6 | 10.8 | (9.5) | (8.5) | (7.7) | (7.0) |
| 50 | 16.89 | 15.7 | 14.8 | (13.3) | (12.1) | (11.2) | (10.5) |
| 75 | 21.65 | 20.1 | 18.9 | (17.1) | (15.7) | (14.7) | (13.9) |
| 100 | 26.77 | 24.7 | 23.1 | 20.8 | 19.2 | 18.0 | (17.1) |
| 125 | 32.28 | 29.4 | 27.3 | 24.3 | 22.3 | 21.0 | (20.1) |
| 150 | 38.27 | 34.2 | 31.3 | 27.5 | 25.2 | 23.7 | (22.7) |
| 175 | 44.99 | 39.1 | 35.2 | 30.5 | 27.7 | 26.0 | (25.0) |
| 200 | 52.95 | 44.2 | 39.0 | 33.2 | 29.8 | 28.1 | (27.1) |
| 225 | 63.23 | 49.9 | 42.7 | 35.7 | 31.8 | 30.0 | (29.2) |
| 250 | 77.90 | 56.9 | 46.8 | 38.2 | 33.9 | 32.1 | (31.4) |
| 275 | 101.06 | 66.3 | 51.6 | 41.0 | 36.2 | 34.5 | (34.2) |
| 300 | 141.64 | 79.7 | 57.9 | 44.4 | 39.2 | 37.7 | (37.8) |
| 325 | 224.59 | 99.4 | 66.6 | 48.8 | 43.2 | 41.9 | (42.4) |
| 350 | 472.54 | 128.7 | 78.3 | 54.7 | 48.4 | 47.3 | (48.2) |
| 375 | | | 93.6 | 62.1 | 54.9 | 53.8 | (55.1) |
| 400 | | | 112.4 | 71.0 | 62.5 | 61.1 | (62.6) |
| 425 | | | 133.7 | 80.9 | 70.1 | 68.0 | (69.5) |
| 450 | | | (155.2) | (90.3) | (76.2) | (72.8) | (73.8) |
| 475 | | | (172.9) | (96.5) | (77.6) | (72.1) | (72.0) |
| 500 | | | (179.3) | (94.3) | (69.4) | (61.0) | (59.2) |
| 525 | | | (161.7) | (76.9) | (44.8) | (32.9) | (28.5) |
| 550 | | | (101.9) | (30.8) | (-5.6) | (-20.8) | (-27.8) |
| 575 | | | (-22.5) | (-56.2) | (-91.4) | (-108.0) | (-117.1) |
| 600 | | | (-233.9) | (-196.0) | (-219.1) | (-232.9) | (-242.2) |

bars where interregional regression discrepancies (Helgeson and Kirkham, 1974a) may introduce higher uncertainties. Uncertainties in the values of the Debye-Hückel parameters involving successive derivatives of ln $\varepsilon$ and ln V increase with differentiation, but errors caused by overfitting the dielectric constant and density data have been minimized (Helgeson and Kirkham, 1974a). Values shown for temperatures $< 100$°C at pressures above a kilobar as well as those for temperatures above 550°C were obtained by extrapolation. The more uncertain values are shown parenthetically in the tables and represented by dashed curves in the figures.

## ACTIVITY COEFFICIENT

Taking account of the ion size parameter (å, which is commonly expressed in angstroms), the Debye-Hückel equation for the mean activity coefficient ($\gamma_\pm$) of a completely dissociated binary electrolyte consisting of $\upsilon$ ions (mole of solute)$^{-1}$ with charges $Z_+$ and $Z_-$ can be written as

$$\log \gamma_\pm = -\frac{|Z_+Z_-|A_\gamma I^{1/2}}{1 + \text{å}B_\gamma I^{1/2}} \tag{1}$$

where I is the ionic strength in molality ($m$) units of concentration ($I = (\upsilon/2)|Z_+Z_-|m$) and $A_\gamma$ and $B_\gamma$ are defined by

$$A_\gamma \equiv \frac{(2\pi N)^{1/2}\, e^3\, \rho^{1/2}}{2.302585(1000)^{1/2}\,(\varepsilon kT)^{3/2}} = \frac{1.824829238 \times 10^6\, \rho^{1/2}}{(\varepsilon T)^{3/2}} \tag{2}$$

and (for eq 1 with å expressed in cm)

$$B_\gamma \equiv \left(\frac{8\pi N \rho e^2}{1000\, \varepsilon kT}\right)^{1/2} = \frac{50.29158649 \times 10^8\, \rho^{1/2}}{(\varepsilon T)^{1/2}} \tag{3}$$

where $\pi = 3.14159265$, $N$ refers to Avogadro's number ($6.02252 \times 10^{23}$ mole$^{-1}$), e stands for the absolute electronic charge (4.80298 esu), $\rho$ is the density in g cm$^{-3}$ and $\varepsilon$ the dielectric constant of $H_2O$, T designates temperature on the thermodynamic scale in °K, and k is Boltzmann's constant ($1.38054 \times 10^{-16}$ erg (°K)$^{-1}$). $A_\gamma$ and $B_\gamma$ in equations (2) and (3) thus have dimensions of kg$^{1/2}$ mole$^{-1/2}$ and kg$^{1/2}$ mole$^{-1/2}$ cm$^{-1}$, respectively.

If $\text{å}B_\gamma$ is set to unity in equation (1), the expression reduces to the Guntelberg (1926) equation, but if $\text{å}B_\gamma$ is taken to be equal to 1.5 the relation becomes the Scatchard (1936) equation. Extension of equation (1) or one of its modifications to high concentrations at low temperatures requires additional terms in an ascending power function of I, such as that used by Lietzke and Stoughton (1962). The simplest of these was proposed originally by Hückel; that is,

$$\log \gamma_\pm = -\frac{|Z_+Z_-|A_\gamma I^{1/2}}{1 + \text{å}B_\gamma I^{1/2}} + b_\gamma I \tag{4}$$

where $b_\gamma$ is an empirical parameter characteristic of the electrolyte. If $\mathring{a}B_\gamma$ is set to unity in equation (4), it reduces to the Guggenheim (1935) equation. Similarly, if $\mathring{a}B_\gamma$ is equated to one and $b_\gamma$ is set to $0.2A_\gamma|Z_+Z_-|$, the expression reduces to the Davies (1938) equation. Owing to changes in $\rho$, $\varepsilon$, and the degree of ion association with increasing temperature and pressure, modified forms of the Debye-Hückel equation and/or its extensions such as the Guntelberg, Scatchard, Guggenheim, and Davies equations are limited in their applicability at high temperatures and pressures.

Values of $A_\gamma$ and $B_\gamma$ computed from equations (2) and (3) are listed in tables 1 and 2 and plotted in figures 2 through 5, where it can be seen that both $A_\gamma$ and $B_\gamma$ decrease to a decreasing degree as pressure increases isothermally (figs. 2 and 4) but increase with increasing temperature to $\sim$ 500°C at constant pressure. At higher temperatures, the isobars in figures 3 and 5 pass through extrema as the density contribution to $A_\gamma$ and $B_\gamma$ becomes more important than the electrostatic contribution. Because the dielectric constant of $H_2O$ decreases as density decreases, either isothermally or isobarically (fig. 1), whether $A_\gamma$ and $B_\gamma$ increase or decrease with increasing temperature and/or pressure depends on the extent to which $(\varepsilon T)^{3/2}$ and $(\varepsilon T)^{1/2}$ increase or decrease relative to the corresponding increase or decrease in $\rho^{1/2}$. The dielectric constant of $H_2O$ decreases by 50 to 70 units as temperature increases isobarically from 0° to $\sim$ 300°C in the liquid phase region below 5 kb, but the corresponding decrease in density is only 0.3 g $cm^{-3}$ or less. Consequently, $A_\gamma$ and $B_\gamma$ increase. In contrast, at high temperatures (and in the steam phase region where the dielectric constant approaches unity at low pressures), the specific volume of $H_2O$ increases dramatically with increasing temperature and/or decreasing pressure, which overcomes the effect of the negative change in the dielectric constant on the temperature and pressure dependence of $A_\gamma$ and $B_\gamma$. As a result, $A_\gamma$ and $B_\gamma$ maximize at high temperatures and decrease with further isobaric increase in temperature. Because $(\varepsilon T)$ is raised to a lower power in equation (3) than in equation (2), the isobars for $B_\gamma$ in figure 5 maximize at slightly lower temperature than those for $A_\gamma$ in figure 3. Although not defined by the calculations, the isotherms for $A_\gamma$ and $B_\gamma$ in figures 2 and 4 should also exhibit maxima at high temperatures and low pressures (of the order of a few hundred bars).

The dielectric constant of $H_2O$ increases by about 10 units as pressure increases isothermally from saturation pressures to $\sim$ 5 kb. Because the corresponding increase in density is slight, the isotherms in figures 2 and 4 decrease monotonically with increasing pressure and appear to approach constant values at high pressures. However, the isotherms may minimize as pressure increases above 5 kb if the rate of increase in the dielectric constant diminishes relative to the rate of increase in density at high pressures.

Perturbation by the critical phenomenon of $A_\gamma$ and $B_\gamma$ as functions of temperature and pressure is evident in figures 6 and 7, where the com-

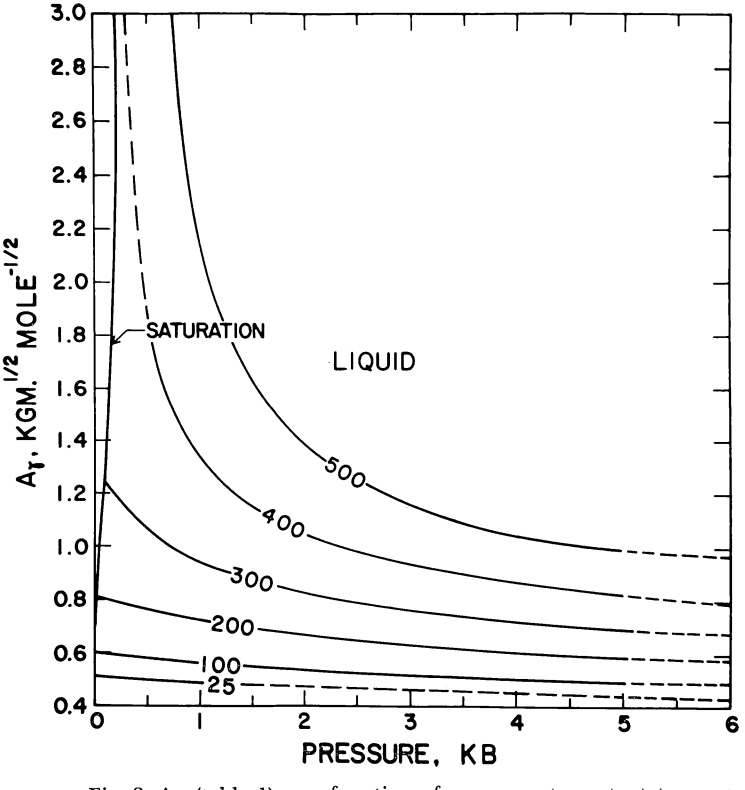


Fig. 2. $A_\gamma$ (table 1) as a function of pressure at constant temperature (labeled in °C) computed from equation (2).

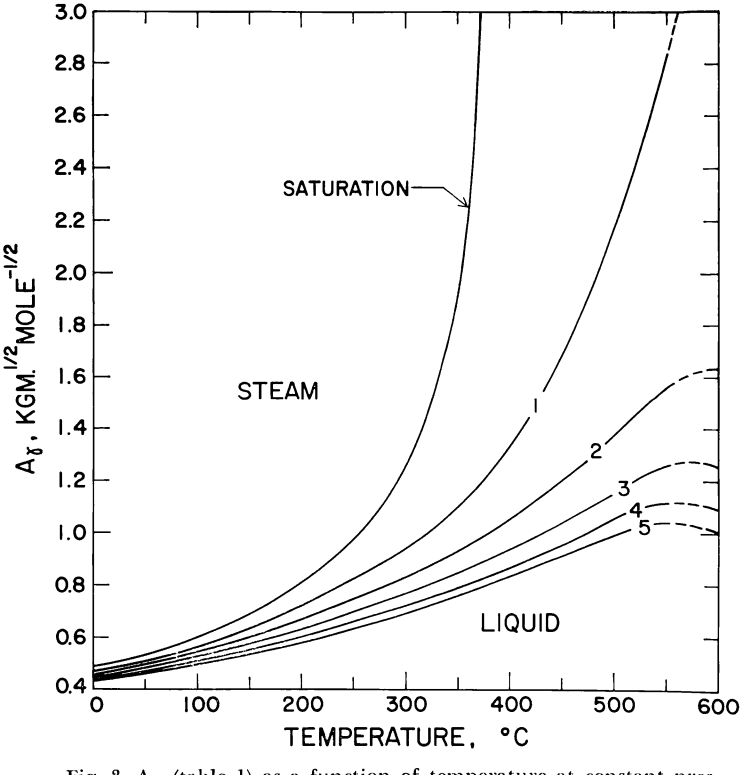


Fig. 3. $A_\gamma$ (table 1) as a function of temperature at constant pressure (labeled in kb) computed from equation (2).

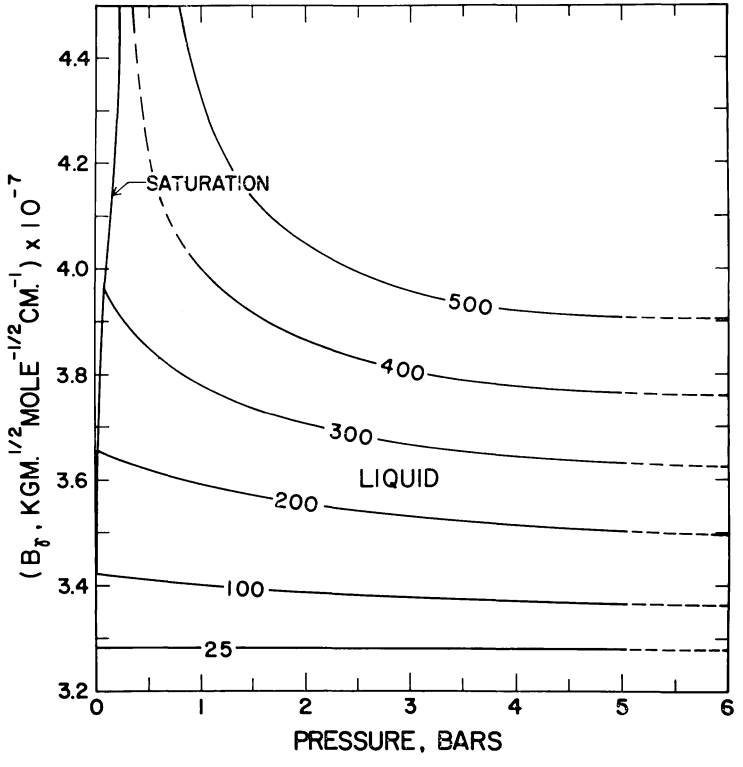


Fig. 4. $B_\gamma$ (table 2) as a function of pressure at constant temperature (labeled in °C) computed from equation (3).

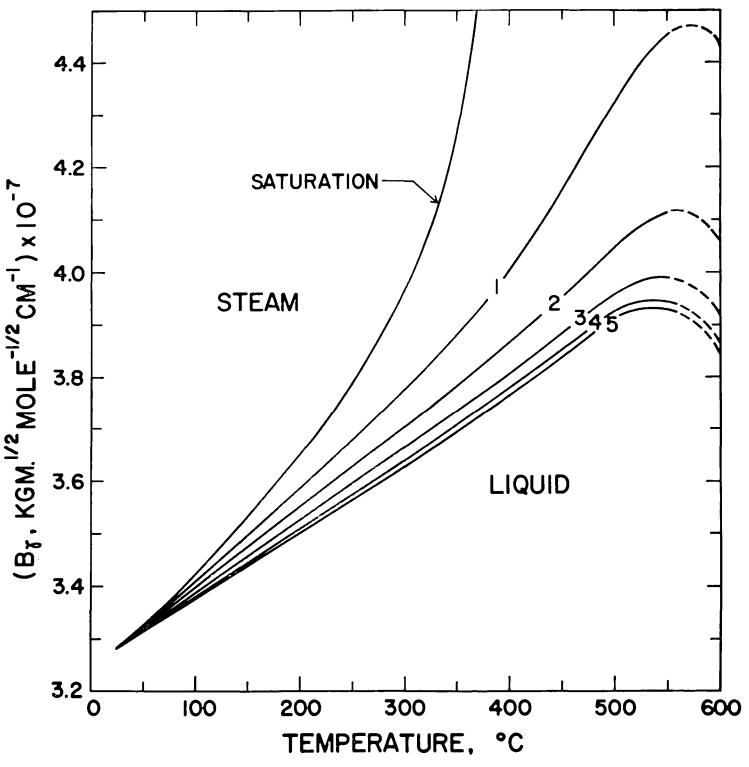


Fig. 5. $B_\gamma$ (table 2) as a function of temperature at constant pressure (labeled in kb) computed from equation (3).

bined effects of increasing temperature and pressure on $A_\gamma$ can be compared with the corresponding effects on $B_\gamma$. It can be seen that the configurations of the isopleths in the two diagrams are comparable. However, because the electrostatic contribution to $A_\gamma$ and $B_\gamma$ dominates the change in these variables over most of the pressure-temperature region considered, $(\partial P/\partial T)_{A\gamma} < (\partial P/\partial T)_{B\gamma}$; that is, as pressure and temperature increase at constant $A_\gamma$, $B_\gamma$ increases. Nevertheless, an increase in temperature from 0° to ~ 300°C along the saturation curve causes corresponding increases in $A_\gamma$ and $B_\gamma$ which are approximately equivalent to those resulting from an increase in temperature from 0° to ~ 500°C at 5 kb.

## ENTHALPY

The partial derivatives of equations (2) and (3) with respect to temperature at constant pressure can be written as

$$\left(\frac{\partial A_\gamma}{\partial T}\right)_P = -\frac{3A_\gamma}{2}\left(\left(\frac{\partial \ln \varepsilon}{\partial T}\right)_P + \frac{1}{T} + \frac{\alpha}{3}\right) \tag{5}$$

and

$$\left(\frac{\partial B_\gamma}{\partial T}\right)_P = -\frac{B_\gamma}{2}\left(\left(\frac{\partial \ln \varepsilon}{\partial T}\right)_P + \frac{1}{T} + \alpha\right) \tag{6}$$

where $\alpha$ stands for the coefficient of isobaric thermal expansion $(-(\partial \ln \rho/\partial T)_P)$. Following the definitions and notation adopted by Lewis and Randall (1961)[1]

$$A_H = 2(2.303)RT^2\left(\frac{\partial A_\gamma}{\partial T}\right)_P \tag{7}$$

and by analogy,

$$B_H = 2(2.303)RT^2\left(\frac{\partial B_\gamma}{\partial T}\right)_P \tag{8}$$

and

$$\mathring{a}_H = 2(2.303)RT^2\left(\frac{\partial \mathring{a}}{\partial T}\right)_P \tag{9}$$

Taking account of equation (1) we can thus write

$$\bar{L}_2 \equiv \bar{H}_2 - \bar{H}^\circ_2 = -2.303\upsilon RT^2\left(\frac{\partial \log \gamma_\pm}{\partial T}\right)_P$$

$$= \frac{\omega A_H I^{1/2}}{1 + \mathring{a}B_\gamma I^{1/2}} - \frac{\omega A_\gamma I(\mathring{a}B_H + B_\gamma \mathring{a}_H)}{(1 + \mathring{a}B_\gamma I^{1/2})^2} \tag{10}$$

[1] For the sake of brevity, 2.303 is shown in equation (7) and subsequent equations, but 2.302585 was used for ln 10 in the calculations.

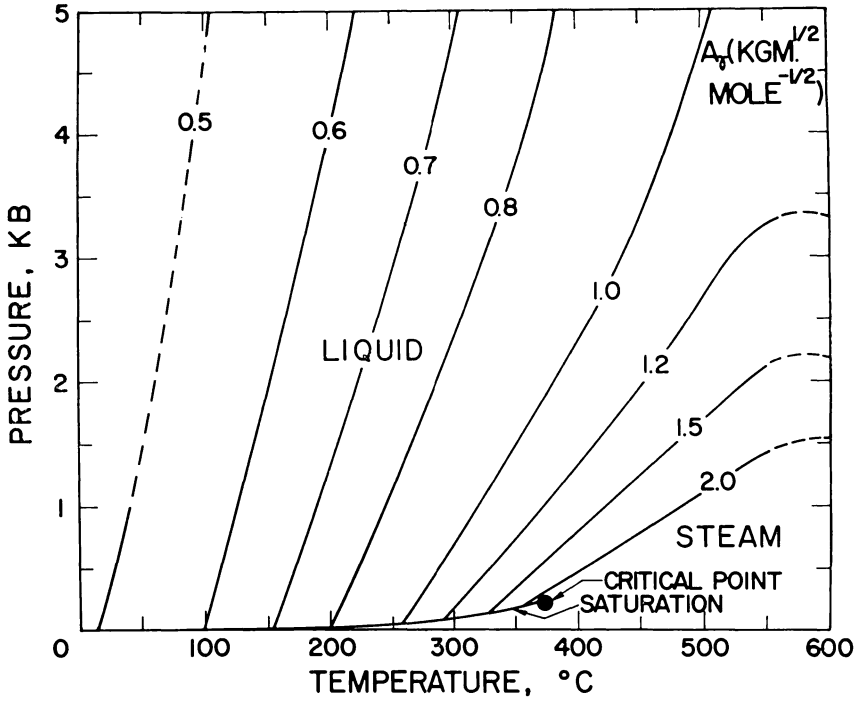


Fig. 6. Isopleths of $A_\gamma$ (labeled in $kg^{1/2}mole^{-1/2}$) as a function of pressure and temperature (table 1 and figs. 2 and 3).

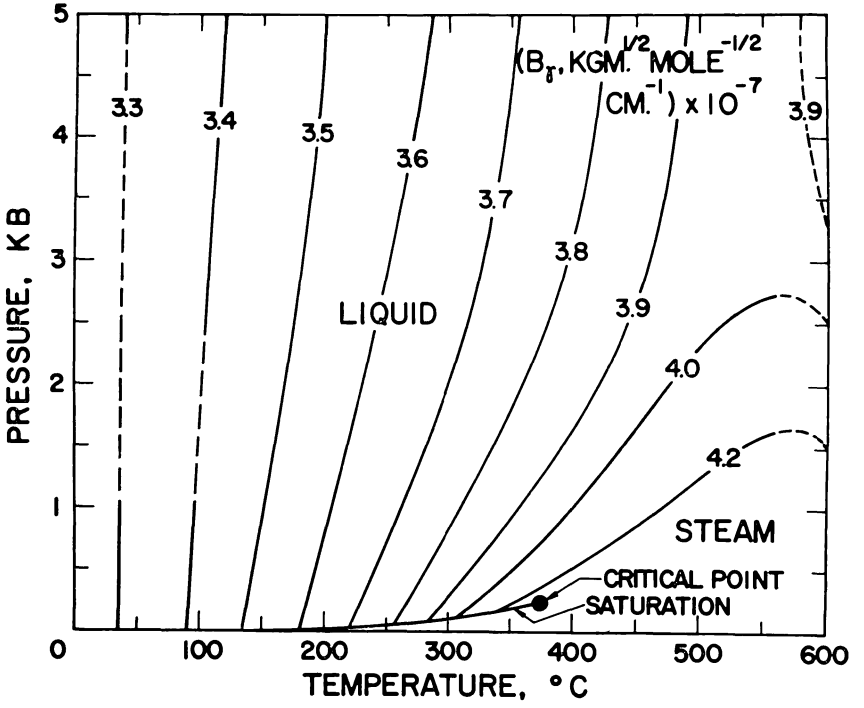


Fig. 7. Isopleths of $B_\gamma$ (labeled in $(kg^{1/2}mole^{-1/2}cm^{-1}) \times 10^{-7}$) as a function of pressure and temperature (table 2 and figs. 4 and 5).

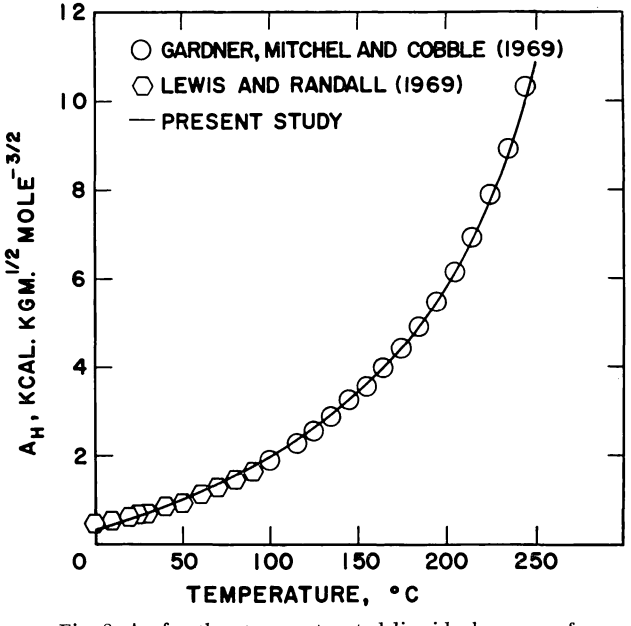


Fig. 8. $A_H$ for the steam-saturated liquid phase as a function of temperature computed (curve) in the present study (table 17) compared with corresponding values taken from the literature.

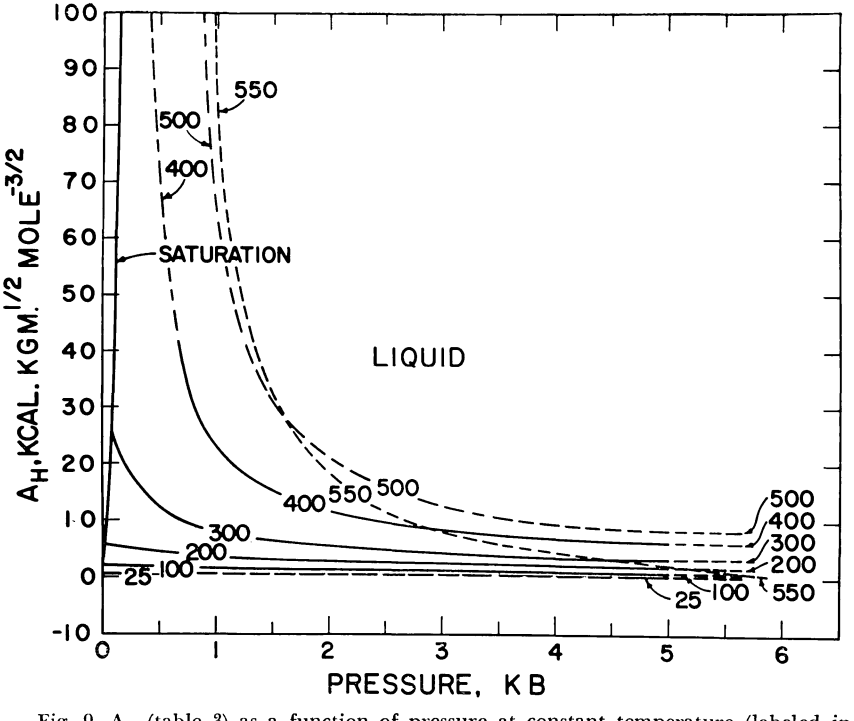


Fig. 9. $A_H$ (table 3) as a function of pressure at constant temperature (labeled in °C) computed from equations (2), (5), and (7).

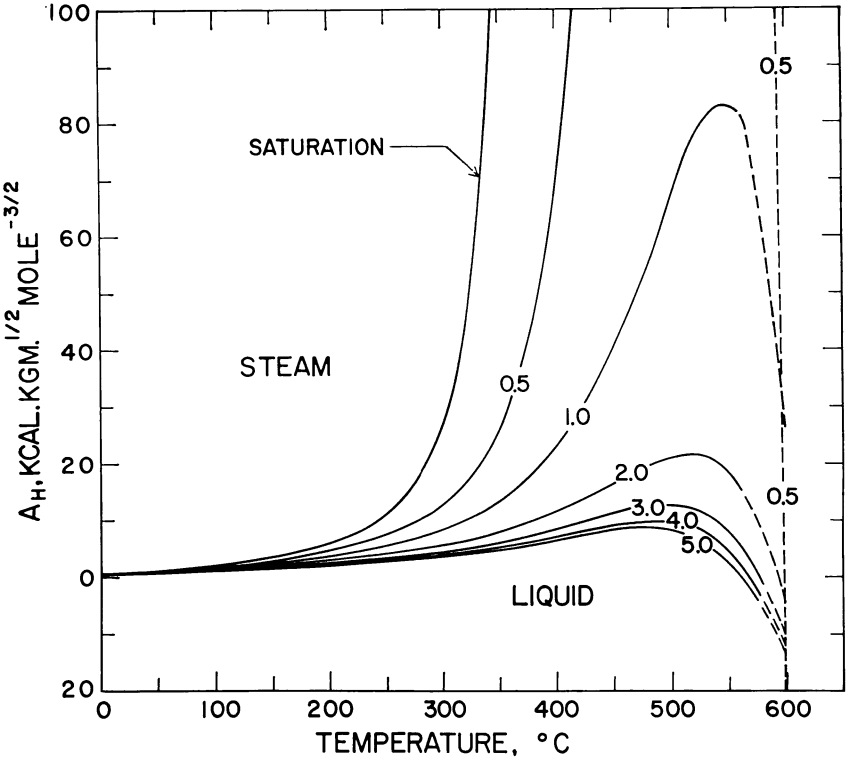


Fig. 10. $A_H$ (table 3) as a function of temperature at constant pressure (labeled in kb) computed from equations (2), (5), and (7).

where $\bar{L}_2$ is the relative partial molal enthalpy of the solute (designated by the subscript 2) at constant temperature and pressure and

$$\omega = \frac{\upsilon|Z_+Z_-|}{2} = \frac{I}{m} \tag{11}$$

Extension of equation (1) to higher concentrations by addition of a $b_\gamma I$ term (eq 4) results in a corresponding term in equation (10) equal to $-\upsilon b_H I/2$, where

$$b_H = 2(2.303)RT^2\left(\frac{\partial b_\gamma}{\partial T}\right)_P \tag{12}$$

If $\mathring{a}B_\gamma$ is then taken to be independent of temperature and set to unity (as in the Guggenheim (1935) modification), it follows that

$$\bar{L}_2 = \frac{\omega A_H I^{1/2}}{1 + I^{1/2}} - \frac{\upsilon b_H I}{2} \tag{13}$$

which is consistent with the equation commonly used to represent the

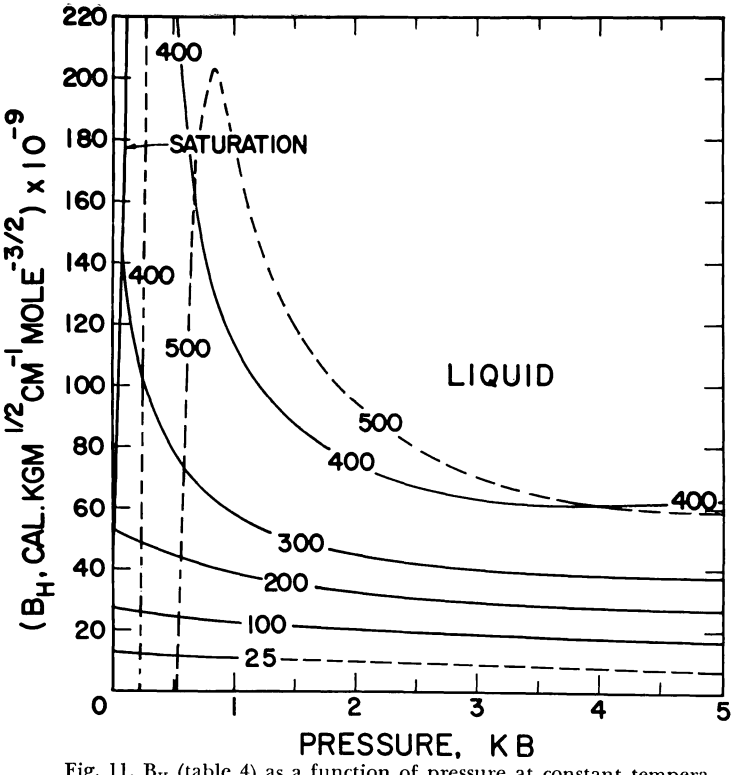


Fig. 11. $B_H$ (table 4) as a function of pressure at constant temperature (labeled in °C) computed from equations (3), (6), and (8).

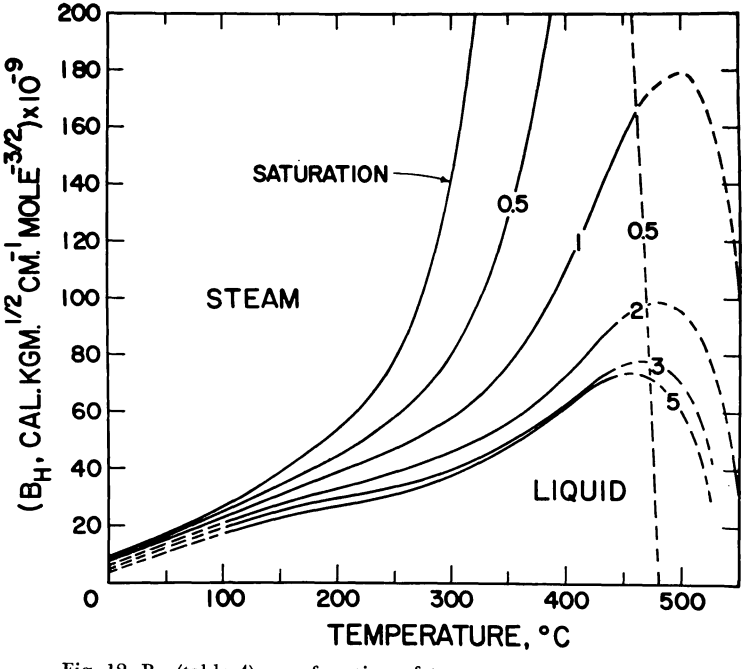


Fig. 12. $B_H$ (table 4) as a function of temperature at constant pressure (labeled in kb) computed from equations (3), (6), and (8).

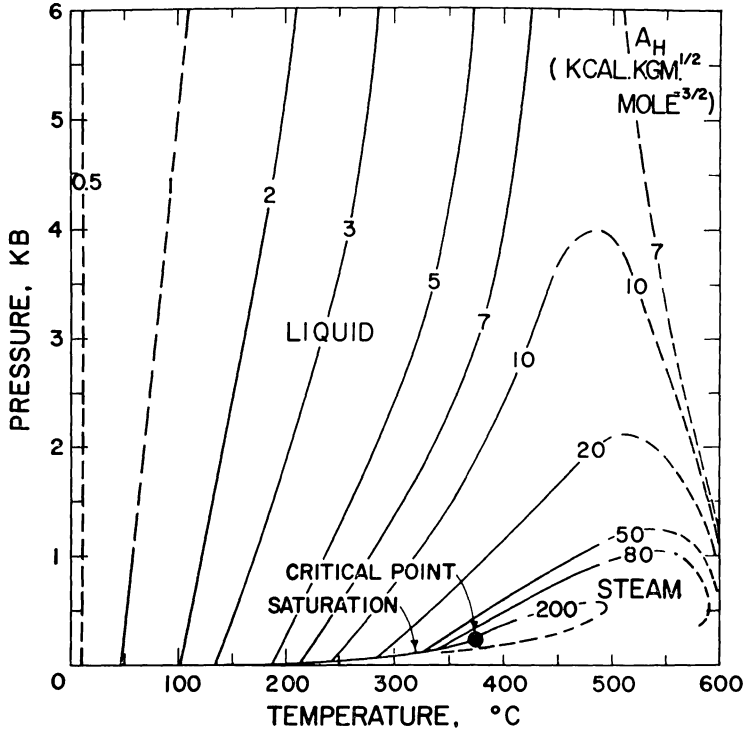


Fig. 13. Isopleths of $A_H$ (labeled in kcal kg$^{1/2}$mole$^{-3/2}$) as a function of pressure and temperature (table 3 and figs. 9 and 10).

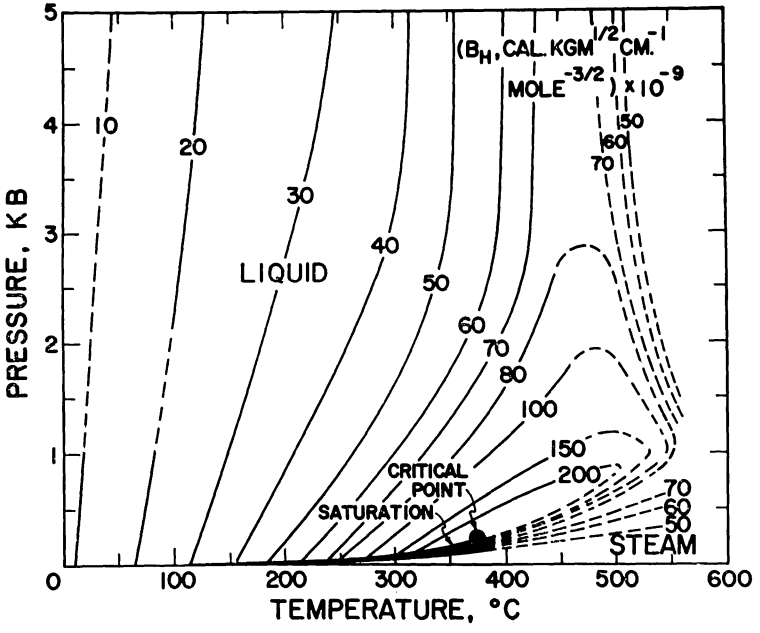


Fig. 14. Isopleths of $B_H$ (labeled in (cal kg$^{1/2}$cm$^{-1}$mole$^{-3/2}$) $\times$ 10$^{-9}$) as a function of pressure and temperature (table 4 and figs. 11 and 12).

TABLE 5

$A_J$ in cal $kg^{1/2}mole^{-3/2}$ $(°K)^{-1}$ computed from equations (2), (5), (16), and (18)—see figures 15 through 17 and 20

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | 13.22 | 12 | 11 | (9) | (8) | (8) | (7) |
| 50 | 15.44 | 14 | 12 | (10) | (9) | (9) | (8) |
| 75 | 18.45 | 16 | 14 | (12) | (10) | (9) | (9) |
| 100 | 22.57 | 19 | 16 | 13 | 11 | 10 | (9) |
| 125 | 28.17 | 22 | 19 | 14 | 12 | 10 | (10) |
| 150 | 35.93 | 27 | 21 | 15 | 12 | 11 | (10) |
| 175 | 47.41 | 32 | 24 | 17 | 13 | 11 | (10) |
| 200 | 66.01 | 40 | 28 | 18 | 14 | 12 | (11) |
| 225 | 99.05 | 52 | 33 | 21 | 15 | 13 | (12) |
| 250 | 163.21 | 71 | 41 | 24 | 17 | 14 | (13) |
| 275 | 302.15 | 103 | 53 | 29 | 20 | 17 | (16) |
| 300 | 660.79 | 158 | 72 | 35 | 25 | 21 | (20) |
| 325 | 1964.42 | 256 | 100 | 44 | 30 | 26 | (24) |
| 350 | 13018.24 | 434 | 140 | 55 | 38 | 32 | (29) |
| 375 | | | 194 | 68 | 46 | 38 | (35) |
| 400 | | | 263 | 83 | 53 | 43 | (38) |
| 425 | | | 347 | 98 | 58 | 44 | (38) |
| 450 | | | (442) | (107) | (55) | (37) | (29) |
| 475 | | | (529) | (103) | (38) | (16) | (6) |
| 500 | | | (546) | (67) | (-6) | (-29) | (-39) |

TABLE 6

$B_J$ in (cal $kg^{1/2}cm^{-1}mole^{-3/2}$ $(°K)^{-1}$) $\times 10^{-8}$ computed from equations (3), (6), (17), and (19)—see figures 18, 19, and 21

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | 1.59 | 1.6 | 1.5 | (1.5) | (1.4) | (1.4) | (1.4) |
| 50 | 1.82 | 1.7 | 1.6 | (1.6) | (1.5) | (1.4) | (1.4) |
| 75 | 1.98 | 1.8 | 1.7 | (1.6) | (1.4) | (1.4) | (1.3) |
| 100 | 2.12 | 1.8 | 1.7 | 1.4 | 1.3 | 1.3 | (1.2) |
| 125 | 2.29 | 1.9 | 1.6 | 1.3 | 1.2 | 1.1 | (1.1) |
| 150 | 2.53 | 1.9 | 1.6 | 1.2 | 1.1 | 1.0 | (1.0) |
| 175 | 2.92 | 2.0 | 1.5 | 1.1 | 0.9 | 0.9 | (0.9) |
| 200 | 3.63 | 2.1 | 1.5 | 1.0 | 0.8 | 0.8 | (0.8) |
| 225 | 5.00 | 2.5 | 1.5 | 1.0 | 0.8 | 0.8 | (0.8) |
| 250 | 7.69 | 3.2 | 1.7 | 1.0 | 0.8 | 0.9 | (1.0) |
| 275 | 13.25 | 4.4 | 2.2 | 1.2 | 1.0 | 1.1 | (1.2) |
| 300 | 26.23 | 6.4 | 2.9 | 1.6 | 1.4 | 1.4 | (1.6) |
| 325 | 67.34 | 9.5 | 4.0 | 2.0 | 1.8 | 1.9 | (2.1) |
| 350 | 353.14 | 14.1 | 5.4 | 2.6 | 2.3 | 2.4 | (2.6) |
| 375 | | | 6.9 | 3.3 | 2.8 | 2.8 | (2.9) |
| 400 | | | 8.1 | 3.8 | 3.1 | 2.9 | (3.0) |
| 425 | | | 8.8 | 4.0 | 2.9 | 2.5 | (2.4) |
| 450 | | | (8.2) | (3.3) | (1.7) | (1.1) | (0.8) |
| 475 | | | (5.5) | (1.2) | (-1.0) | (-2.0) | (-2.5) |
| 500 | | | (-1.2) | (-3.5) | (-6.1) | (-7.4) | (-8.2) |

TABLE 7

$A_E$ in kcal $kg^{1/2}mole^{-3/2}$ computed from equations (2), (26), and (28)—see figures 22 through 24 and 27

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | 0.582 | 0.52 | 0.47 | (0.40) | (0.35) | (0.32) | (0.29) |
| 50 | 0.773 | 0.70 | 0.64 | (0.56) | (0.50) | (0.46) | (0.42) |
| 75 | 0.972 | 0.88 | 0.81 | (0.71) | (0.65) | (0.60) | (0.56) |
| 100 | 1.167 | 1.06 | 0.97 | 0.86 | 0.78 | 0.73 | (0.69) |
| 125 | 1.346 | 1.21 | 1.11 | 0.98 | 0.90 | 0.85 | (0.81) |
| 150 | 1.493 | 1.33 | 1.22 | 1.08 | 1.00 | 0.95 | (0.92) |
| 175 | 1.591 | 1.40 | 1.28 | 1.15 | 1.07 | 1.04 | (1.02) |
| 200 | 1.623 | 1.42 | 1.29 | 1.17 | 1.12 | 1.11 | (1.12) |
| 225 | 1.569 | 1.36 | 1.25 | 1.17 | 1.16 | 1.18 | (1.22) |
| 250 | 1.409 | 1.22 | 1.15 | 1.14 | 1.19 | 1.27 | (1.35) |
| 275 | 1.115 | 1.01 | 1.02 | 1.11 | 1.25 | 1.39 | (1.53) |
| 300 | 0.632 | 0.72 | 0.87 | 1.11 | 1.35 | 1.57 | (1.77) |
| 325 | -0.179 | 0.34 | 0.72 | 1.17 | 1.53 | 1.84 | (2.11) |
| 350 | -1.801 | -0.14 | 0.62 | 1.32 | 1.81 | 2.22 | (2.57) |
| 375 | | | 0.58 | 1.58 | 2.20 | 2.71 | (3.14) |
| 400 | | | 0.65 | 1.96 | 2.70 | 3.29 | (3.80) |
| 425 | | | 0.83 | 2.41 | 3.23 | 3.89 | (4.48) |
| 450 | | | (1.09) | (2.78) | (3.65) | (4.37) | (5.02) |
| 475 | | | (1.19) | (2.78) | (3.66) | (4.41) | (5.13) |
| 500 | | | (0.44) | (1.79) | (2.74) | (3.58) | (4.36) |
| 525 | | | (-2.98) | (-1.20) | (0.12) | (1.16) | (2.06) |
| 550 | | | (-12.96) | (-7.68) | (-5.19) | (-3.64) | (-2.50) |
| 575 | | | (-36.14) | (-19.39) | (-14.12) | (-11.46) | (-9.82) |
| 600 | | | (-79.58) | (-37.31) | (-26.88) | (-22.26) | (-19.74) |

TABLE 8

$B_E$ in (cal $kg^{1/2}cm^{-1}mole^{-3/2}$) $\times 10^{-9}$ computed from equations (3), (27), and (29)—see figures 25, 26, and 28

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | 12.51 | 11.5 | 10.6 | (9.4) | (8.4) | (7.7) | (7.1) |
| 50 | 16.06 | 14.9 | 14.0 | (12.6) | (11.6) | (10.8) | (10.3) |
| 75 | 19.37 | 18.1 | 17.0 | (15.5) | (14.4) | (13.7) | (13.2) |
| 100 | 22.20 | 20.7 | 19.5 | 17.9 | 16.9 | 16.2 | (15.7) |
| 125 | 24.31 | 22.6 | 21.4 | 19.7 | 18.7 | 18.2 | (17.9) |
| 150 | 25.49 | 23.7 | 22.4 | 20.8 | 20.0 | 19.7 | (19.7) |
| 175 | 25.57 | 23.7 | 22.4 | 21.2 | 20.7 | 20.8 | (21.1) |
| 200 | 24.41 | 22.5 | 21.5 | 20.8 | 20.9 | 21.5 | (22.3) |
| 225 | 21.96 | 20.4 | 19.8 | 19.8 | 20.8 | 22.1 | (23.6) |
| 250 | 18.20 | 17.2 | 17.3 | 18.6 | 20.6 | 22.9 | (25.2) |
| 275 | 13.13 | 13.3 | 14.5 | 17.3 | 20.8 | 24.3 | (27.6) |
| 300 | 6.66 | 8.8 | 11.6 | 16.6 | 21.7 | 26.5 | (31.0) |
| 325 | -1.63 | 3.8 | 9.1 | 16.7 | 23.6 | 30.0 | (35.8) |
| 350 | -13.27 | -1.4 | 7.3 | 17.9 | 26.8 | 34.9 | (42.1) |
| 375 | | | 6.4 | 20.5 | 31.3 | 41.0 | (49.7) |
| 400 | | | 6.6 | 24.1 | 36.8 | 48.0 | (58.0) |
| 425 | | | 7.7 | 28.1 | 42.1 | 54.6 | (65.9) |
| 450 | | | (9.0) | (30.6) | (45.5) | (58.8) | (71.1) |
| 475 | | | (8.8) | (28.8) | (43.6) | (57.2) | (70.0) |
| 500 | | | (2.9) | (17.5) | (31.2) | (44.6) | (57.4) |
| 525 | | | (-17.4) | (-11.1) | (1.4) | (14.0) | (26.3) |
| 550 | | | (-67.6) | (-67.3) | (-55.0) | (-42.9) | (-31.4) |
| 575 | | | (-171.1) | (-163.3) | (-147.0) | (-133.9) | (-122.9) |
| 600 | | | (-353.0) | (-308.2) | (-279.9) | (-262.5) | (-250.7) |

apparent molal enthalpy of the solute at constant temperature and pressure ($\phi_H$) as a function of ionic strength; that is,

$$\phi_H = \bar{H}^\circ_2 + \omega A_H I^{1/2}\left( (1 + I^{1/2})^{-1} - \frac{\sigma(I^{1/2})}{3}\right) - \frac{b_H \upsilon I}{4} \qquad (14)$$

where

$$\sigma(I^{1/2}) = \frac{3}{I^{3/2}}\left( 1 + I^{1/2} - \frac{1}{1 + I^{1/2}} - 2 \ln (1 + I^{1/2}) \right) \qquad (15)$$

The term $\upsilon b_H I/2$ in equation (13) is numerically equivalent to Lewis and Randall's (1961) $2(2.303)RT^2 (\partial B/\partial T)_P\, m\upsilon_+\upsilon_-$.

Values of $A_H$ and $B_H$ computed from equations (5) through (8) are given in tables 3 and 4 and plotted in figures 8 through 12. $A_H$ values computed by Lewis and Randall (1961) and Gardner, Mitchell, and Cobble (1969) can be compared with those generated in this study for temperatures from 0° to 250°C in figure 8. The curve in figure 8 is in close agreement with the values calculated by Gardner, Mitchell, and Cobble but departs systematically to a slight extent from the low temperature values of Lewis and Randall (1961), which are based on the dielectric constant data of Malmberg and Maryott (1956) and densities taken from Dorsey (1940).

As expected from the isobaric behavior of $A_\gamma$ and $B_\gamma$ as functions of temperature in figures 3 and 5, the isobars in figures 10 and 12 pass through extrema at high temperatures, and the temperature corresponding to the maximum maximizes in the vicinity of a kilobar as pressure increases from the steam region to 5 kb. Note in figure 11 that the high temperature isotherms maximize at low pressures. With increasing pressure above ~ a kilobar at constant temperature, $A_H$ and $B_H$ decrease monotonically and approach constant values at high pressures.

The dependence of $A_H$ and $B_H$ on temperature and pressure is largely controlled by the behavior and relative magnitude of $\alpha$ compared to $(\partial \ln \varepsilon/\partial T)_P$, which are of opposite sign and approach $\infty$ and $-\infty$, respectively, at the critical point. The coefficient of isobaric thermal expansion increases at low temperatures with increasing temperature at constant pressure and the isobars maximize at or above the critical temperature. In contrast, $(\partial \ln \varepsilon/\partial T)_P$ becomes less negative with increasing temperature at constant pressure at both low and high temperatures, owing to isobaric extrema in the vicinity of 200° to 300°C and the minimum at the critical point (Helgeson and Kirkham, 1974a). As a consequence, $(\partial \ln \varepsilon/\partial T)_P$ controls the behavior of $A_H$ and $B_H$ at low temperatures, but $\alpha$ dominates the behavior of these functions at high temperatures (especially at low pressures), which causes $A_H$ and $B_H$ to become negative at high temperatures. Accordingly, the isopleths in figures 13 and 14 wrap around the critical point in a configuration similar to that exhibited by corresponding isopleths for $\alpha$. Because $\alpha$ is divided by 3 in equation (5), the contours

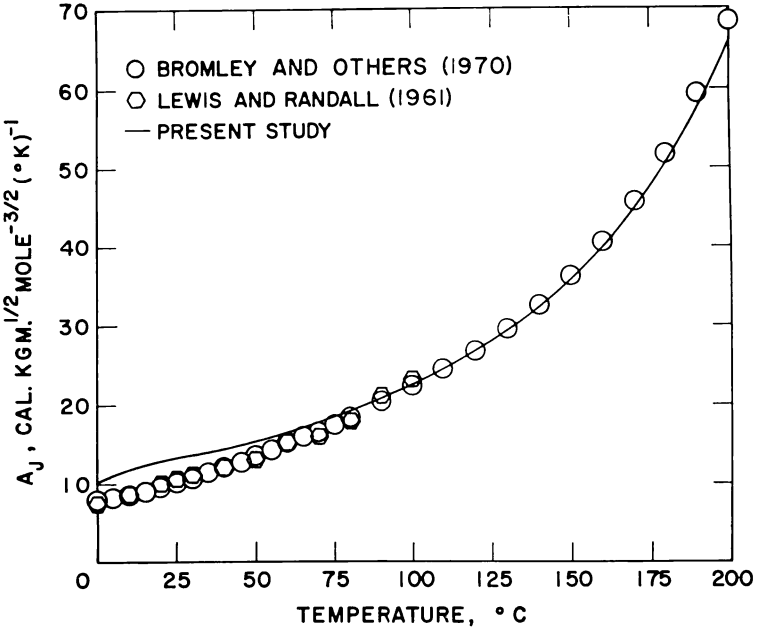


Fig. 15. $A_J$ for the steam-saturated liquid phase as a function of temperature computed (curve) in the present study (table 17) compared with corresponding values taken from the literature.

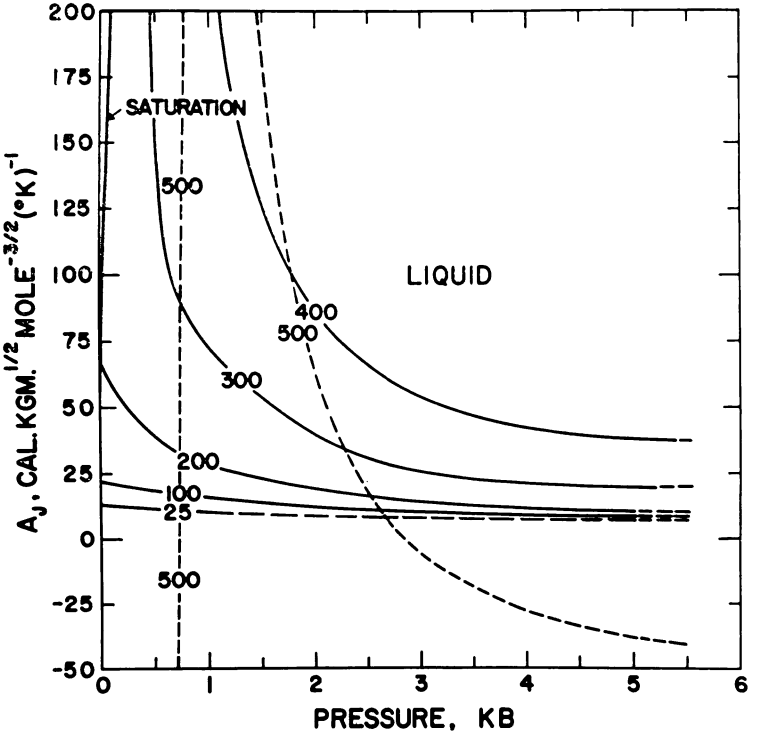


Fig. 16. $A_J$ (table 5) as a function of pressure at constant temperature (labeled in °C) computed from equations (2), (5), (16), and (18).

around the critical point in figure 18 are not as convoluted as they are in figure 19.

It can be deduced from figures 9 through 14 that $A_H$ and $B_H$ as functions of temperature and pressure describe positive saddle-shaped asymptotic surfaces about the critical region. Nevertheless, in the immediate vicinity of the critical point, $A_H$ and $B_H$ should decrease and approach $-3(2.303)RT\ A_\gamma$ and $-2.303RT\ B_\gamma$, respectively. The configuration described by isopleths of $A_H$ and $B_H$ in the critical region should thus be similar to that defined by the contours of a precipitous volcano with a deep central vent (not shown in figs. 13 and 14).

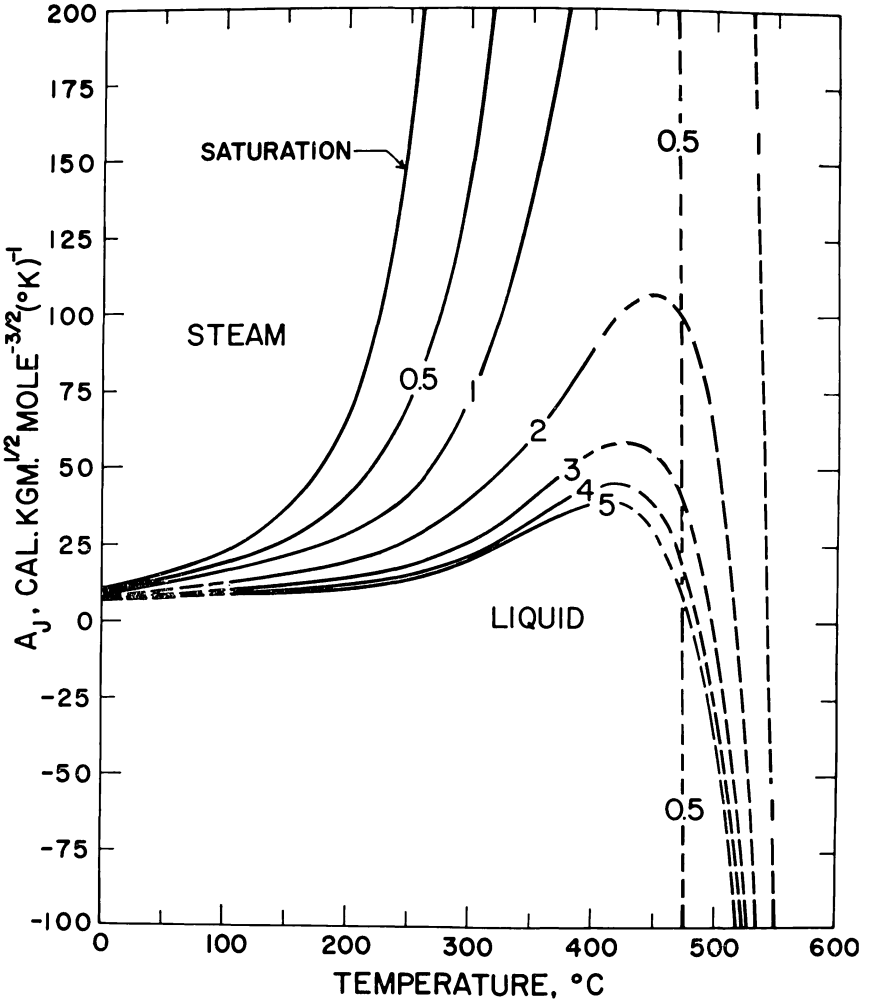


Fig. 17. $A_J$ (table 5) as a function of temperature at constant pressure (labeled in kb) computed from equations (2), (5), (16), and (18).

HEAT CAPACITY AT CONSTANT PRESSURE

Differentiating equations (7) and (8) with respect to temperature at constant pressure leads to

$$A_J = \left(\frac{\partial A_H}{\partial T}\right)_P = 2(2.303)R\left(2T\left(\frac{\partial A_\gamma}{\partial T}\right)_P + T^2\left(\frac{\partial^2 A_\gamma}{\partial T^2}\right)_P\right)$$

$$= \frac{2A_H}{T} + 2(2.303)RT^2\left(\frac{\partial^2 A_\gamma}{\partial T^2}\right)_P \tag{16}$$

and

$$B_J = \left(\frac{\partial B_H}{\partial T}\right)_P = 2(2.303)R\left(2T\left(\frac{\partial B_\gamma}{\partial T}\right)_P + T^2\left(\frac{\partial^2 B_\gamma}{\partial T^2}\right)_P\right)$$

$$= \frac{2B_H}{T} + 2(2.303)RT^2\left(\frac{\partial^2 A_\gamma}{\partial T^2}\right)_P \tag{17}$$

where (from eqs 5 and 6)

$$\left(\frac{\partial^2 A_\gamma}{\partial T^2}\right)_P = \frac{1}{A_\gamma}\left(\frac{\partial A_\gamma}{\partial T}\right)_P^2 - \frac{3A_\gamma}{2}\left(\left(\frac{\partial^2 \ln \varepsilon}{\partial T^2}\right)_P - \frac{1}{T^2} + \frac{1}{3}\left(\frac{\partial \alpha}{\partial T}\right)_P\right) \tag{18}$$

and

$$\left(\frac{\partial^2 B_\gamma}{\partial T^2}\right)_P = \frac{1}{B_\gamma}\left(\frac{\partial B_\gamma}{\partial T}\right)_P^2 - \frac{B_\gamma}{2}\left(\left(\frac{\partial^2 \ln \varepsilon}{\partial T^2}\right) - \frac{1}{T^2} + \left(\frac{\partial \alpha}{\partial T}\right)_P\right) \tag{19}$$

The relative partial molal heat capacity of the solute at constant pressure ($\bar{J}_2$) consistent with equations (1) and (10) can thus be expressed as

$$\bar{J}_2 = \bar{C}_{P,2} - \bar{C}^\circ_{P,2} = \left(\frac{\partial \bar{L}_2}{\partial T}\right)_P$$

$$= 2.303\upsilon R\left(2T\left(\frac{\partial \log \gamma_\pm}{\partial T}\right)_P + T^2\left(\frac{\partial^2 \log \gamma_\pm}{\partial T^2}\right)_P\right)$$

$$= \frac{\omega A_J I^{1/2}}{1 + \mathring{a}B_\gamma I^{1/2}} - \frac{\omega I(A_H(\mathring{a}B_H + B_\gamma \mathring{a}_H) + A_\gamma \mathring{a}_H B_H)}{2.303RT^2(1 + \mathring{a}B_\gamma I^{1/2})^2}$$

$$- \frac{\omega A_\gamma I(\mathring{a}B_J + B_\gamma \mathring{a}_J)}{(1 + \mathring{a}B_\gamma I^{1/2})^2} + \frac{\omega A_\gamma I^{3/2}(\mathring{a}B_H + B_\gamma \mathring{a}_H)^2}{2.303RT^2(1 + \mathring{a}B_\gamma I^{1/2})^3} \tag{20}$$

or from equation (13) as

$$\bar{J}_2 = \frac{\omega A_J I^{1/2}}{1 + I^{1/2}} - \frac{\upsilon b_J I}{2} \tag{21}$$

which is consistent with

$$\phi_{C_P} = C^\circ_{P,2} + \omega A_J I^{1/2}\left((1 + I^{1/2})^{-1} - \frac{\sigma(I^{1/2})}{3}\right) - \frac{\upsilon b_J I}{4} \qquad (22)$$

where $\phi_{C_P}$ is the apparent molal heat capacity of the solute at constant temperature and pressure,

$$\mathring{a}_J = \left(\frac{\partial \mathring{a}_H}{\partial T}\right)_P = 2(2.303)R\left(2T\left(\frac{\partial \mathring{a}}{\partial T}\right)_P + T^2\left(\frac{\partial^2 \mathring{a}}{\partial T^2}\right)_P\right)$$
$$= \frac{2\mathring{a}_H}{T} + 2(2.303)RT^2\left(\frac{\partial^2 \mathring{a}}{\partial T^2}\right)_P \qquad (23)$$

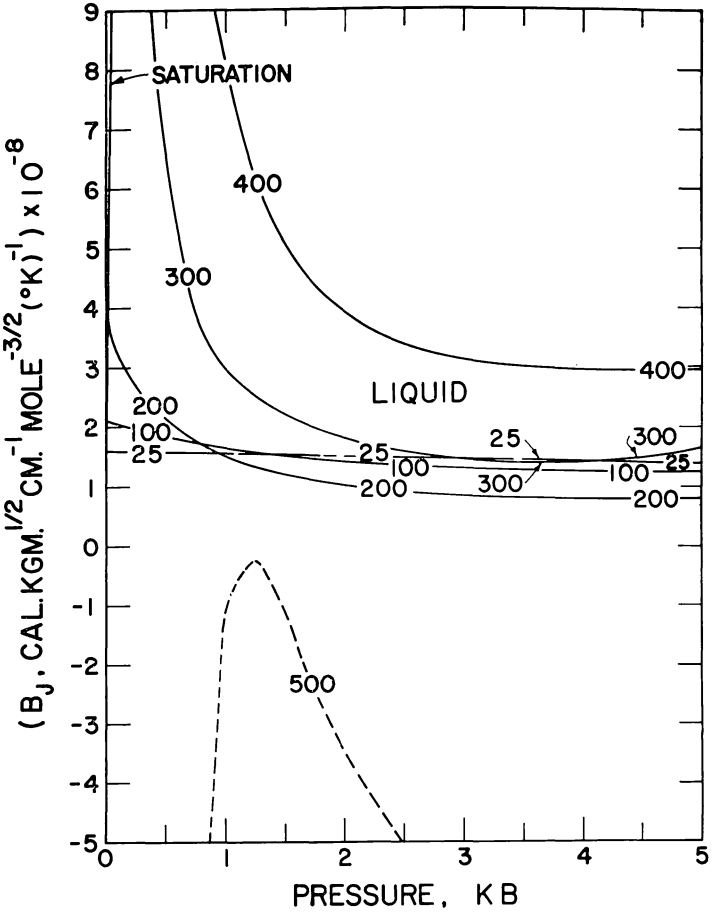


Fig. 18. $B_J$ (table 6) as a function of pressure at constant temperature (labeled in °C) computed from equations (3), (6), (17), and (19).

and

$$b_J = \left(\frac{\partial b_H}{\partial T}\right)_P = 2(2.303)R\left(2T\left(\frac{\partial b_\gamma}{\partial T}\right)_P + T^2\left(\frac{\partial^2 b_\gamma}{\partial T^2}\right)_P\right)$$

$$= \frac{2b_H}{T} + 2(2.303)RT^2\left(\frac{\partial^2 b_\gamma}{\partial T^2}\right)_P \qquad (24)$$

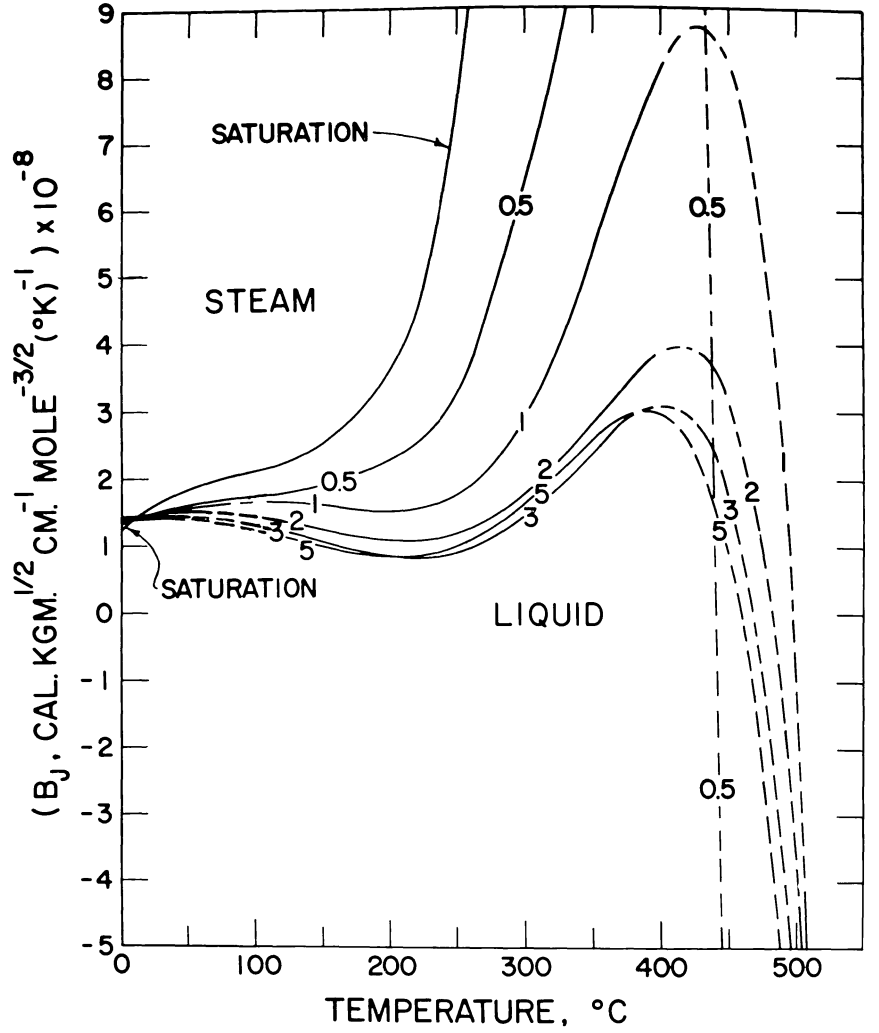


Fig. 19. $B_J$ (table 6) as a function of temperature at constant pressure (labeled in kb) computed from equations (3), (6), (17), and (19).

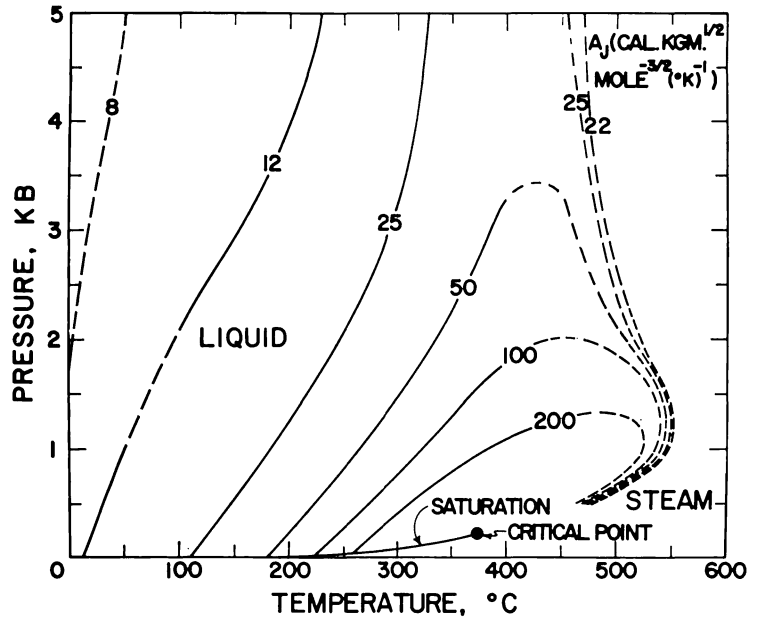


Fig. 20. Isopleths of $A_J$ (labeled in cal $kg^{1/2}mole^{-3/2}(°K)^{-1}$) as a function of pressure and temperature (table 5 and figs. 16 and 17).

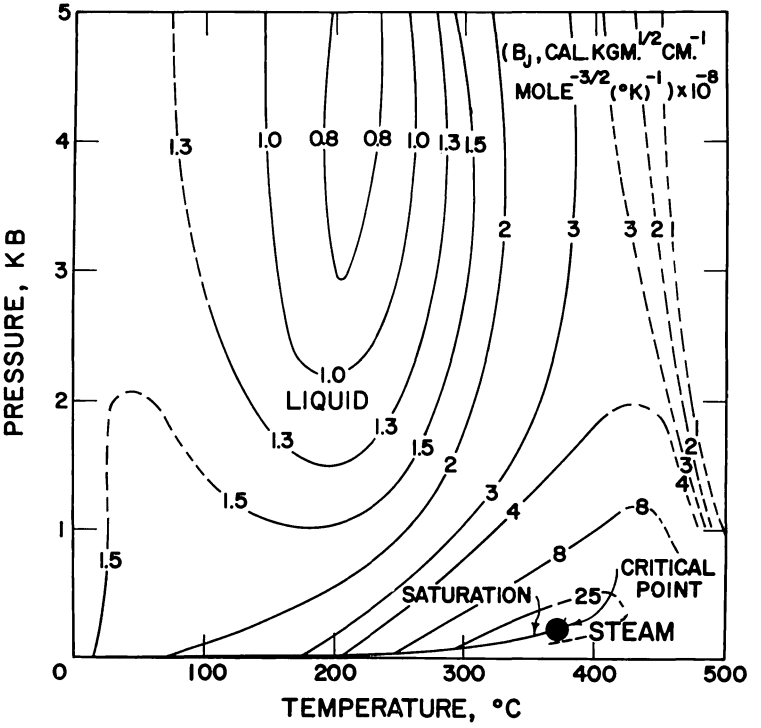


Fig. 21. Isopleths of $B_J$ (labeled in (cal $kg^{1/2}cm^{-1}mole^{-3/2}(°K)^{-1}$) $\times 10^{-8}$) as a function of pressure and temperature (table 6 and figs. 18 and 19).

The term $\upsilon b_J I/4$ in equation (22), which corresponds to the partial derivative of equation (14), is numerically equivalent to $2.303RT^2\ m\upsilon_+\upsilon_-(2(\partial B/\partial T)_P/T + (\partial^2 B/\partial T^2)_P)$ in the comparable equation given by Lewis and Randall (1961).

Values of $A_J$ and $B_J$ computed from equations (16) through (19) are given in tables 5 and 6 and plotted in figures 15 through 19. It can be seen in figure 15 that the values of $A_J$ computed in this study are in close agreement with those reported by Bromley and others (1970) and Lewis and Randall (1961) from 90° to 200°C along the saturation curve, but at lower temperatures the curve in figure 15 departs to higher values of $A_J$ than those reported in the literature. The discrepancies result from the high sensitivity of $A_J$ to $(\partial^2 \ln \varepsilon/\partial T^2)_P$ and the fact that the low temperature values taken from the literature were obtained by fitting dielectric constants reported by Malmberg and Maryott (1956) in contrast to the curve in figure 15, which is based on composite regression of data given by Owen and others (1961), Oshry (ms), and Heger (ms). The curve in figure 15 is consistent with that in figure 8, but Malmberg and Maryott's (1956) data are not closely consistent with the values of the dielectric constant reported by Oshry (ms) at higher temperatures (Helgeson and Kirkham, 1974a).

As expected from the behavior of the isobars in figures 10 and 12, those in figures 17 and 19 exhibit extrema between the temperature corresponding to the critical point and 450°C. Note also that the temperatures represented by the extrema maximize with increasing pressure. It

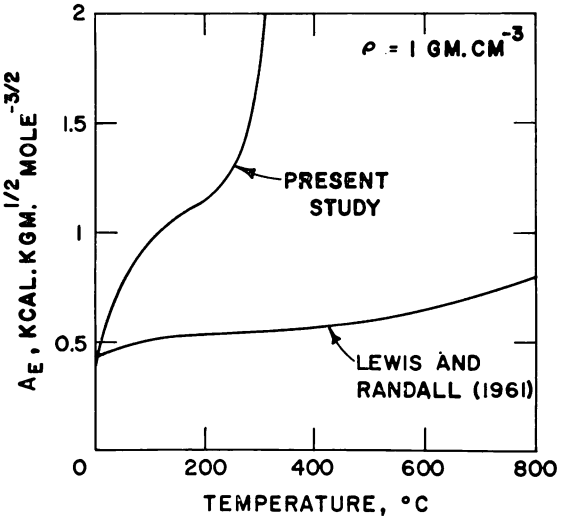


Fig. 22. $A_E$ for a density of $H_2O$ of one g $cm^{-3}$ computed in the present study compared with corresponding values taken from the literature.

can be seen in figures 16 and 18 that both $A_J$ and $B_J$ maximize with increasing pressure at constant temperature $\gtrsim$ the critical temperature at pressures ranging from the critical pressure to $\sim$ a kilobar at 500°C. Note that the isopleths in figures 20 and 21 wrap around the critical point, as they do for $A_H$ and $B_H$ in figures 13 and 14. In contrast to those for $\alpha$, $\beta$, and $C_P$ of $H_2O$ (which approach infinity at the critical point), the isopleths for $A_J$ and $B_J$ in the immediate vicinity of the critical point should describe a "volcanic vent" configuration (not shown in figs. 20 and 21) like (but offset from) those of $A_H$ and $B_H$. It can be seen in figure 21 that the isopleths for $B_J$ minimize at $\sim$ 200°C and high pressures. No corresponding minima in the $A_J$ isopleths are apparent in figure 20.

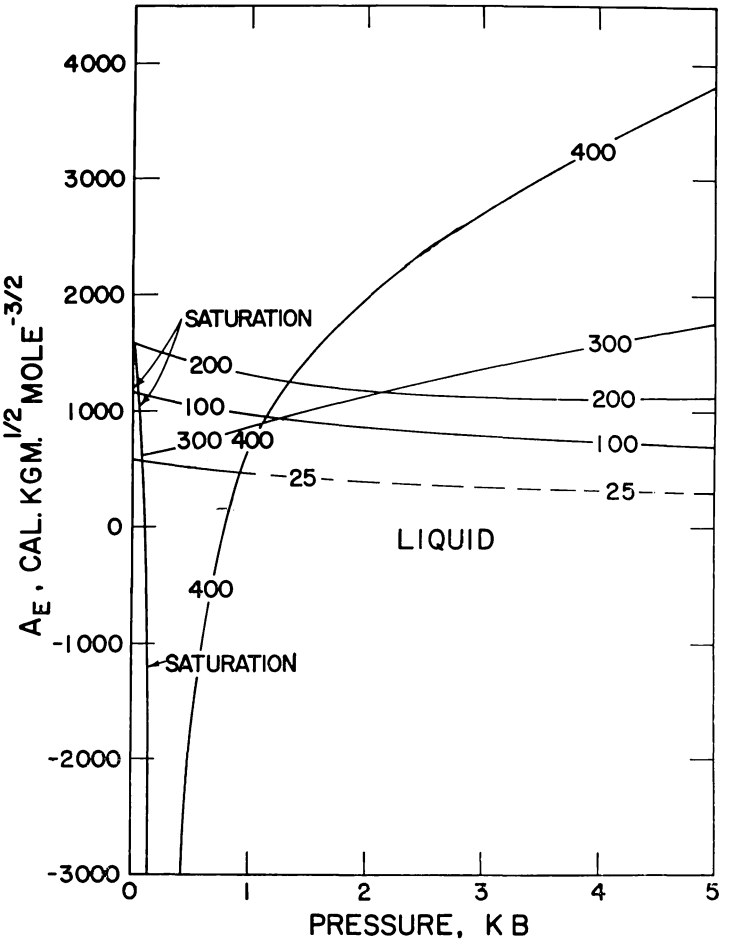


Fig. 23. $A_E$ (table 7) as a function of pressure at constant temperature (labeled in °C) computed from equations (2), (26), and (28).

INTERNAL ENERGY

The Debye-Hückel parameters for the relative partial molal internal energy of the solute at constant *temperature* and *volume* ($\bar{E}_2 - \bar{E}^\circ_2$) defined by

$$\bar{E}_2 - \bar{E}^\circ_2 = -2.303_v RT^2 \left(\frac{\partial \log \gamma_\pm}{\partial T}\right)_\rho \tag{25}$$

can be expressed as

$$A_E = 2(2.303)RT^2 \left(\frac{\partial A_\gamma}{\partial T}\right)_\rho \tag{26}$$

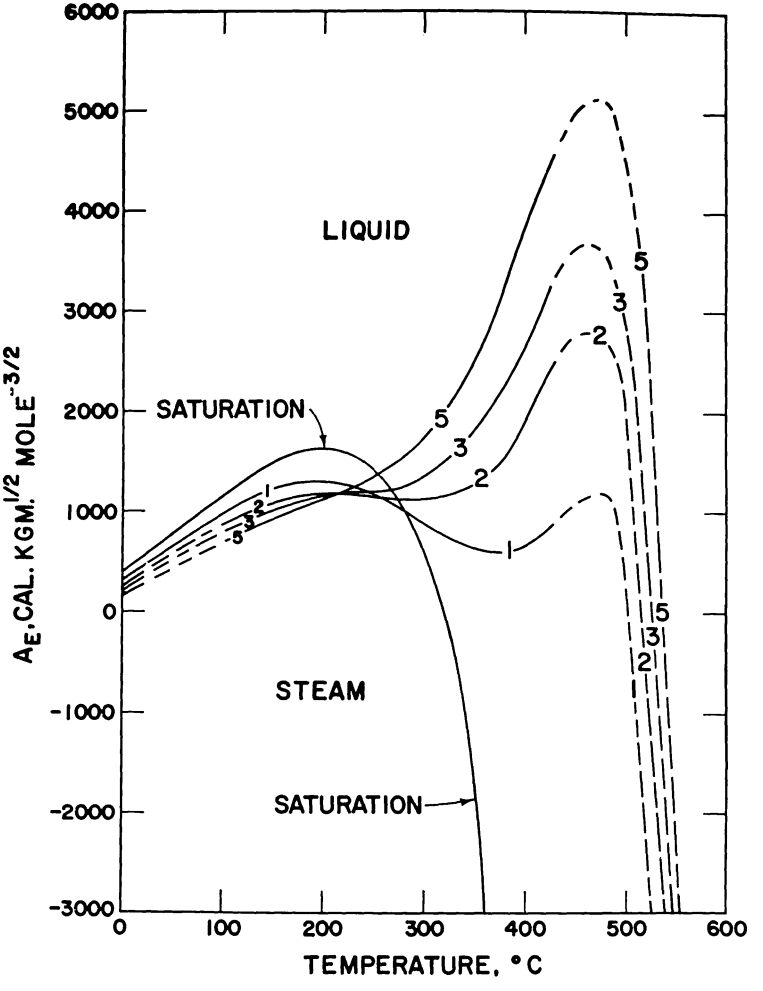


Fig. 24. $A_E$ (table 7) as a function of temperature at constant pressure (labeled in kb) computed from equations (2), (26), and (28).

and

$$B_E = 2(2.303)RT^2\left(\frac{\partial B_\gamma}{\partial T}\right)_\rho \tag{27}$$

where (from eqs 2and 3)

$$\left(\frac{\partial A_\gamma}{\partial T}\right)_\rho = -\frac{3A_\gamma}{2}\left(\left(\frac{\partial \ln \varepsilon}{\partial T}\right)_\rho + \frac{1}{T}\right) \tag{28}$$

and

$$\left(\frac{\partial B_\gamma}{\partial T}\right)_\rho = -\frac{B_\gamma}{2}\left(\left(\frac{\partial \ln \varepsilon}{\partial T}\right)_\rho + \frac{1}{T}\right) = \frac{B_\gamma}{3A_\gamma}\left(\frac{\partial A_\gamma}{\partial T}\right)_\rho \tag{29}$$

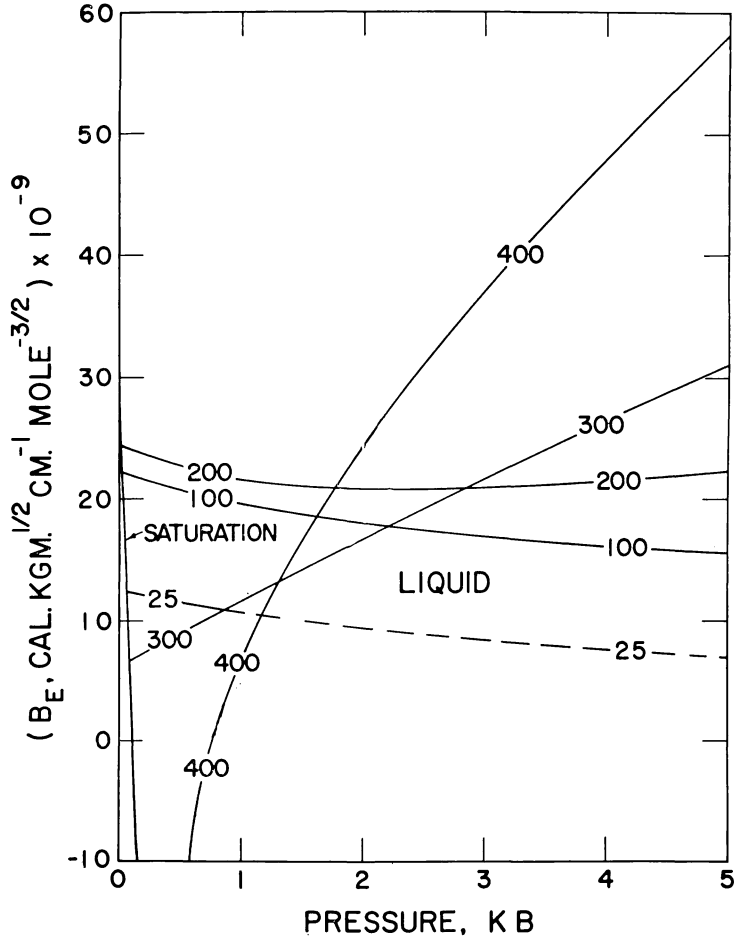


Fig. 25. $B_E$ (table 8) as a function of pressure at constant temperature (labeled in °C) computed from equations (3), (27), and (29).

Hence

$$B_E = \frac{A_E B_\gamma}{3A_\gamma} \tag{30}$$

The internal energy analogs of equations (10), (13), and (14) for constant temperature and volume can now be written as

$$\bar{E}_2 - \bar{E}^\circ_2 = \frac{\omega A_E I^{1/2}}{1 + \mathring{a}B_\gamma I^{1/2}} - \frac{\omega A_\gamma I(\mathring{a}B_E + B_\gamma \mathring{a}_E)}{(1 + \mathring{a}B_\gamma I^{1/2})^2} \tag{31}$$

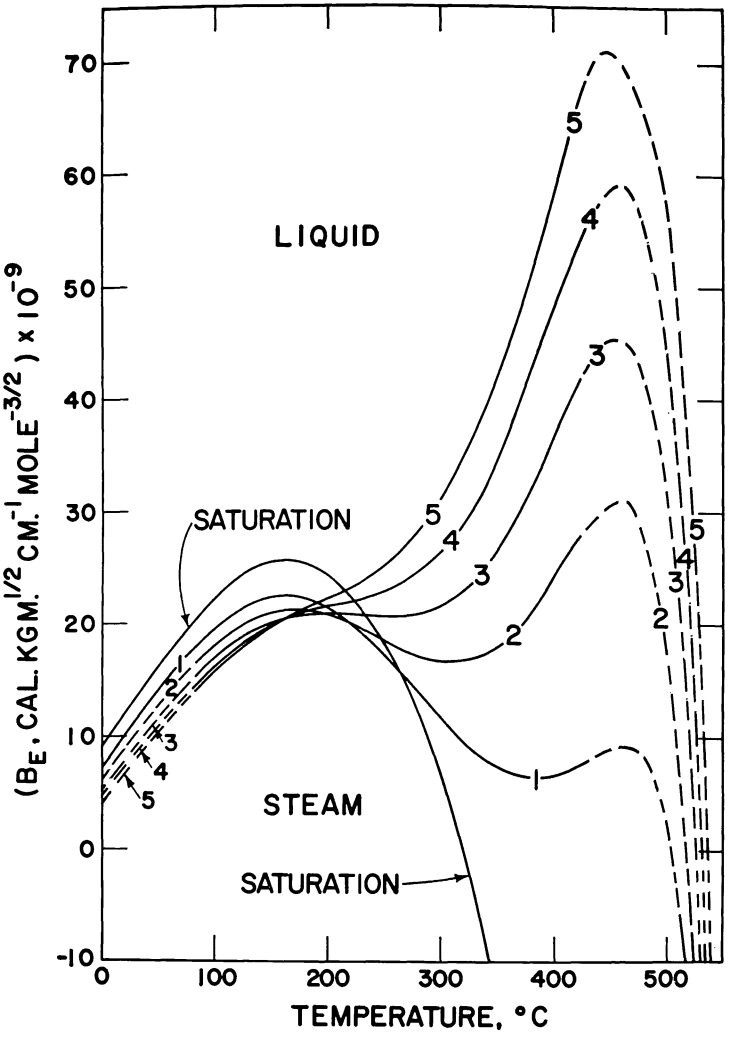


Fig. 26. $B_E$ (table 8) as a function of temperature at constant pressure (labeled in kb) computed from equations (3), (27), and (29).

TABLE 9

$A_{C_V}$ in cal $kg^{1/2}mole^{-3/2}$ $(°K)^{-1}$ computed from equations (2), (28), (37), and (39)—see figures 29 through 31 and 34

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | 6.61 | 6.1 | 5.8 | (5.3) | (5.0) | (4.9) | (4.8) |
| 50 | 6.20 | 5.8 | 5.5 | (5.1) | (4.9) | (4.8) | (4.8) |
| 75 | 5.29 | 5.0 | 4.7 | (4.5) | (4.5) | (4.5) | (4.6) |
| 100 | 3.75 | 3.6 | 3.6 | 3.6 | 3.8 | 4.0 | (4.3) |
| 125 | 1.52 | 1.7 | 1.9 | 2.4 | 3.0 | 3.5 | (3.9) |
| 150 | -1.42 | -0.7 | 0.0 | 1.1 | 2.1 | 2.9 | (3.7) |
| 175 | -4.99 | -3.4 | -2.2 | -0.2 | 1.4 | 2.6 | (3.7) |
| 200 | -9.02 | -6.2 | -4.1 | -1.2 | 1.1 | 2.8 | (4.3) |
| 225 | -13.26 | -8.7 | -5.5 | -1.5 | 1.5 | 3.8 | (5.6) |
| 250 | -17.44 | -10.3 | -5.9 | -0.8 | 3.0 | 5.8 | (8.0) |
| 275 | -21.32 | -10.8 | -4.8 | 1.3 | 5.7 | 8.9 | (11.4) |
| 300 | -24.85 | -9.5 | -2.0 | 5.0 | 9.7 | 13.1 | (15.9) |
| 325 | -28.50 | -6.0 | 3.0 | 10.3 | 14.9 | 18.5 | (21.5) |
| 350 | -35.41 | 0.5 | 10.0 | 16.7 | 20.9 | 24.4 | (27.5) |
| 375 | | | 18.8 | 23.5 | 26.8 | 30.1 | (33.3) |
| 400 | | | 28.5 | 29.1 | 31.1 | 33.9 | (37.1) |
| 425 | | | 36.5 | 30.5 | 30.9 | 33.2 | (36.4) |
| 450 | | | (36.6) | (22.5) | (21.8) | (23.7) | (26.9) |
| 475 | | | (15.2) | (-3.4) | (-3.3) | (-0.8) | ( 2.5) |
| 500 | | | (-56.1) | (-60.0) | (-53.6) | (-48.4) | (-44.1) |

TABLE 10

$B_{C_V}$ in (cal $kg^{1/2}cm^{-1}mole^{-3/2}$ $(°K)^{-1}$) $\times 10^{-8}$ computed from equations (3), (29), (38), and (40)—see figures 32, 33, and 35

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | 1.30 | 1.2 | 1.2 | (1.2) | (1.1) | (1.1) | (1.1) |
| 50 | 1.13 | 1.1 | 1.1 | (1.0) | (1.0) | (1.1) | (1.1) |
| 75 | 0.85 | 0.8 | 0.8 | (0.9) | (0.9) | (0.9) | (1.0) |
| 100 | 0.49 | 0.5 | 0.5 | 0.6 | 0.7 | 0.8 | (0.9) |
| 125 | 0.04 | 0.1 | 0.2 | 0.3 | 0.5 | 0.6 | (0.7) |
| 150 | -0.47 | -0.3 | -0.2 | 0.1 | 0.3 | 0.5 | (0.6) |
| 175 | -1.00 | -0.7 | -0.5 | -0.2 | 0.1 | 0.4 | (0.6) |
| 200 | -1.52 | -1.1 | -0.8 | -0.3 | 0.1 | 0.4 | (0.7) |
| 225 | -1.97 | -1.4 | -1.0 | -0.4 | 0.2 | 0.6 | (0.9) |
| 250 | -2.32 | -1.5 | -0.9 | -0.2 | 0.4 | 0.9 | (1.3) |
| 275 | -2.54 | -1.2 | -0.7 | 0.1 | 0.9 | 1.4 | (1.9) |
| 300 | -2.62 | -0.7 | -0.3 | 0.7 | 1.5 | 2.1 | (2.6) |
| 325 | -2.60 | 0.0 | -0.4 | 1.4 | 2.2 | 2.9 | (3.4) |
| 350 | -2.63 | | 1.2 | 2.2 | 3.0 | 3.6 | (4.2) |
| 375 | | | 2.0 | 3.0 | 3.7 | 4.3 | (4.9) |
| 400 | | | 2.9 | 3.5 | 4.1 | 4.6 | (5.2) |
| 425 | | | 3.3 | 3.5 | 3.8 | 4.3 | (4.8) |
| 450 | | | (3.0) | (2.4) | (2.5) | (2.8) | (3.3) |
| 475 | | | (1.1) | (-0.4) | (-0.6) | (-0.4) | (-0.1) |
| 500 | | | (-3.7) | (-5.9) | (-6.2) | (-6.2) | (-6.1) |

TABLE 11

$A_V$ in $cm^3kg^{1/2}mole^{-3/2}$ computed from equations (2), (48), and (51)—see figures 36 through 38 and 41

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | 2.75 | 2.37 | 2.13 | (1.67) | (1.35) | (1.10) | (0.90) |
| 50 | 3.43 | 2.94 | 2.65 | (2.09) | (1.67) | (1.35) | (1.11) |
| 75 | 4.48 | 3.77 | 3.33 | (2.58) | (2.04) | (1.65) | (1.35) |
| 100 | 6.03 | 4.93 | 4.24 | 3.21 | 2.49 | 2.00 | (1.63) |
| 125 | 8.29 | 6.52 | 5.43 | 4.01 | 3.04 | 2.41 | (1.95) |
| 150 | 11.60 | 8.72 | 7.01 | 5.04 | 3.71 | 2.89 | (2.31) |
| 175 | 16.56 | 11.77 | 9.08 | 6.34 | 4.51 | 3.44 | (2.72) |
| 200 | 24.21 | 16.06 | 11.83 | 7.97 | 5.48 | 4.08 | (3.17) |
| 225 | 36.56 | 22.22 | 15.51 | 9.99 | 6.62 | 4.79 | (3.65) |
| 250 | 57.81 | 31.34 | 20.47 | 12.45 | 7.94 | 5.58 | (4.17) |
| 275 | 97.83 | 45.34 | 27.29 | 15.43 | 9.48 | 6.46 | (4.72) |
| 300 | 184.17 | 67.85 | 36.83 | 19.03 | 11.25 | 7.43 | (5.31) |
| 325 | 419.46 | 106.14 | 50.45 | 23.41 | 13.30 | 8.52 | (5.94) |
| 350 | 1504.78 | 176.05 | 70.18 | 28.79 | 15.68 | 9.73 | (6.62) |
| 375 | | | 99.06 | 35.46 | 18.48 | 11.11 | (7.36) |
| 400 | | | 141.49 | 43.87 | 21.79 | 12.70 | (8.20) |
| 425 | | | 203.66 | 54.58 | 25.76 | 14.56 | (9.15) |
| 450 | | | (294.23) | (68.29) | (30.57) | (16.76) | (10.27) |
| 475 | | | (424.14) | (85.86) | (36.39) | (19.36) | (11.58) |
| 500 | | | (603.15) | (108.10) | (43.38) | (22.43) | (13.14) |
| 525 | | | (830.08) | (131.18) | (52.05) | (27.27) | (14.96) |
| 550 | | | (1080.52) | (160.58) | (61.08) | (31.20) | (16.92) |
| 575 | | | (1305.01) | (190.49) | (70.13) | (35.06) | (18.88) |
| 600 | | | (1439.53) | (215.40) | (77.59) | (38.17) | (20.51) |

TABLE 12

$B_V$ in $(cm^2kg^{1/2}mole^{-3/2}) \times 10^{-6}$ computed from equations (3), (49), and (52)—see figures 39, 40, and 42

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | -2.07 | -2.15 | | | | | |
| 50 | -10.32 | -9.08 | -8.16 | (-5.98) | (-4.07) | (-2.49) | (-1.13) |
| 75 | -20.47 | -17.43 | -15.34 | (-11.31) | (-8.07) | (-5.59) | (-3.54) |
| 100 | -33.92 | -27.99 | -23.99 | -17.47 | -12.52 | -8.97 | (-6.11) |
| 125 | -52.24 | -41.59 | -34.59 | -24.74 | -17.52 | -12.61 | (-8.80) |
| 150 | -77.61 | -59.23 | -47.68 | -33.41 | -23.14 | -16.48 | (-11.54) |
| 175 | -113.39 | -82.31 | -63.89 | -43.72 | -29.40 | -20.52 | (-14.21) |
| 200 | -165.21 | -112.85 | -84.02 | -55.81 | -36.27 | -24.61 | (-16.68) |
| 225 | -243.18 | -153.94 | -109.20 | -69.71 | -43.67 | -28.62 | (-18.80) |
| 250 | -366.91 | -232.08 | -140.98 | -85.42 | -51.48 | -32.42 | (-20.43) |
| 275 | -578.63 | -327.84 | -181.59 | -102.90 | -59.55 | -35.85 | (-21.43) |
| 300 | -984.09 | -475.22 | -234.24 | -122.21 | -67.77 | -38.82 | (-21.68) |
| 325 | -1926.64 | -715.90 | -303.34 | -143.56 | -76.09 | -41.26 | (-21.13) |
| 350 | -5336.98 | -1142.27 | -394.50 | -167.43 | -84.57 | -43.18 | (-19.76) |
| 375 | | | -514.09 | -194.60 | -93.43 | -44.68 | (-17.61) |
| 400 | | | -668.02 | -226.23 | -103.08 | -46.01 | (-14.85) |
| 425 | | | -860.20 | -263.90 | -114.17 | -47.56 | (-11.76) |
| 450 | | | -1090.55 | -309.61 | -127.63 | -49.92 | (-8.78) |
| 475 | | | (-1350.36) | (-365.70) | (-144.62) | (-53.85) | (-6.51) |
| 500 | | | (-1612.73) | (-434.46) | (-166.41) | (-60.24) | (-5.73) |
| 525 | | | (-1825.03) | (-500.56) | (-195.74) | (-73.44) | (-7.30) |
| 550 | | | (-1924.25) | (-586.25) | (-228.70) | (-87.15) | (-11.85) |
| 575 | | | (-1871.34) | (-670.91) | (-264.95) | (-103.97) | (-19.51) |
| 600 | | | (-1652.62) | (-737.60) | (-298.70) | (-121.51) | (-29.37) |

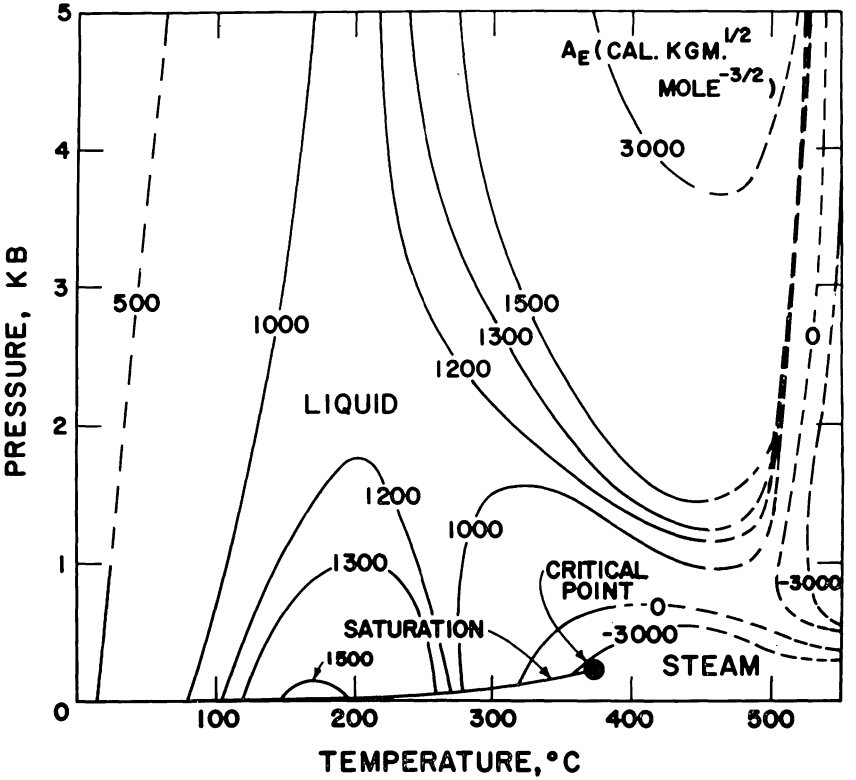


Fig. 27. Isopleths of $A_E$ (labeled in cal kg$^{1/2}$mole$^{-3/2}$) as a function of pressure and temperature (table 7 and figs. 23 and 24).

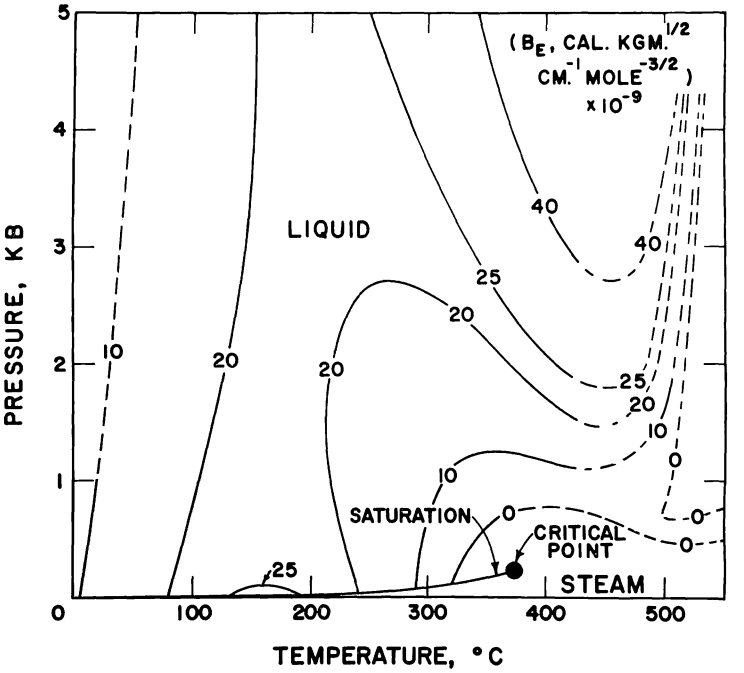


Fig. 28. Isopleths of $B_E$ (labeled in (cal kg$^{1/2}$cm$^{-1}$mole$^{-3/2}$) $\times$ $10^{-9}$) as a function of pressure and temperature (table 8 and figs. 25 and 26).

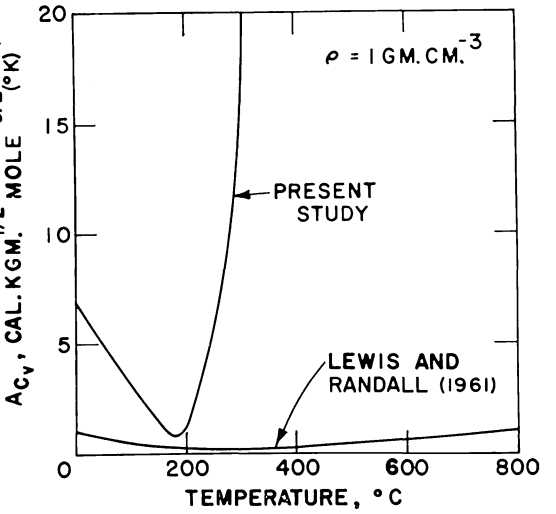


Fig. 29. $A_{C_V}$ for a density of $H_2O$ of one g cm$^{-3}$ computed in the present study compared with corresponding values taken from the literature.

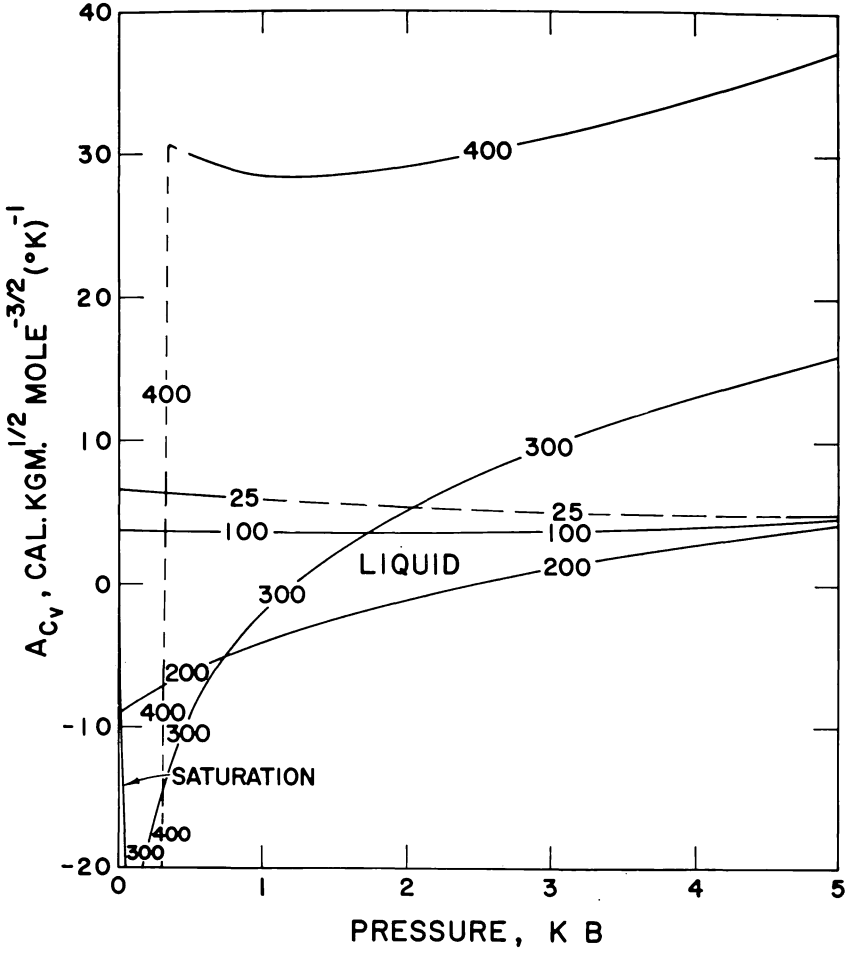


Fig. 30. $A_{C_V}$ (table 9) as a function of pressure at constant temperature (labeled in °C) computed from equations (2), (28), (37), and (39).

$$\bar{E}_2 - \bar{E}^\circ_2 = \frac{\omega A_E I^{1/2}}{1 + I^{1/2}} - \frac{\upsilon b_E I}{2} \tag{32}$$

and

$$\phi_E = \bar{E}^\circ_2 + \omega A_E I^{1/2}\left( (1+ I^{1/2})^{-1} - \frac{\sigma(I^{1/2})}{3}\right) - \frac{b_E \upsilon I}{4} \tag{33}$$

where $\phi_E$ is the apparent molal internal energy of the solute at constant temperature and volume,

$$\mathring{a}_E = 2(2.303)RT^2\left(\frac{\partial \mathring{a}}{\partial T}\right)_\rho \tag{34}$$

and

$$b_E = 2(2.303)RT^2\left(\frac{\partial b_\gamma}{\partial T}\right)_\rho \tag{35}$$

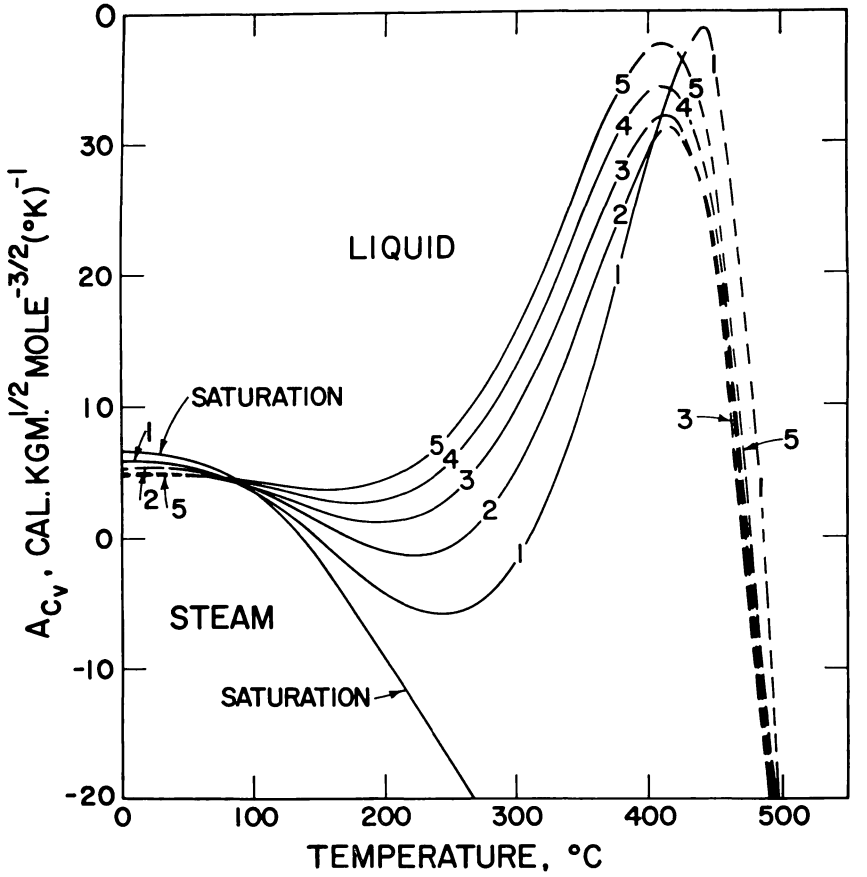


Fig. 31. $A_{C_V}$ (table 9) as a function of temperature at constant pressure (labeled in kb) computed from equations (2), (28), (37), and (39).

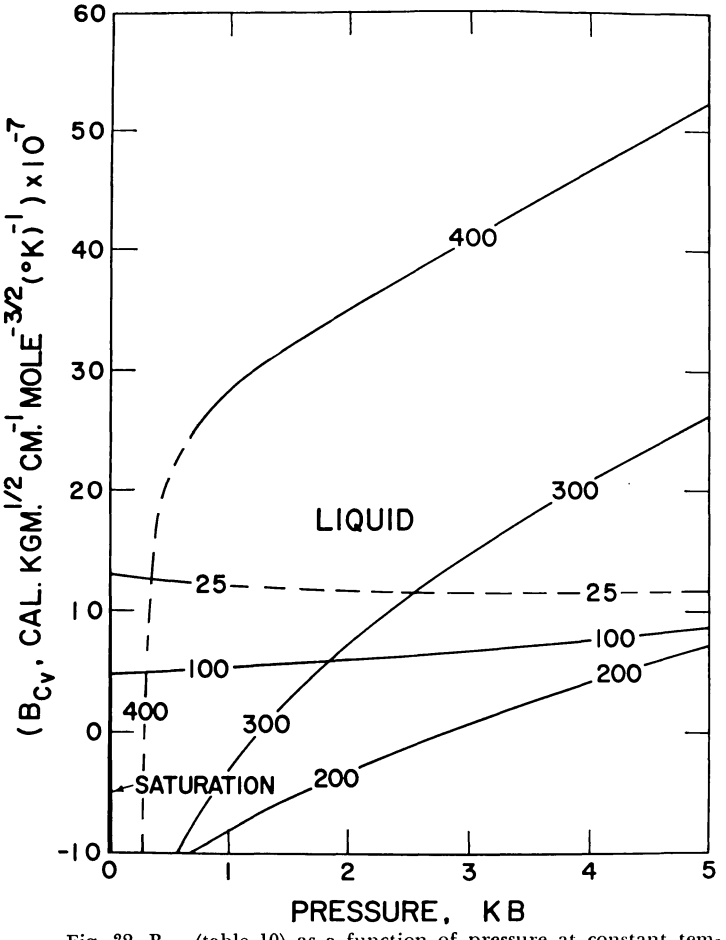


Fig. 32. $B_{cv}$ (table 10) as a function of pressure at constant temperature (labeled in °C) computed from equations (3), (29), (38), and (40).

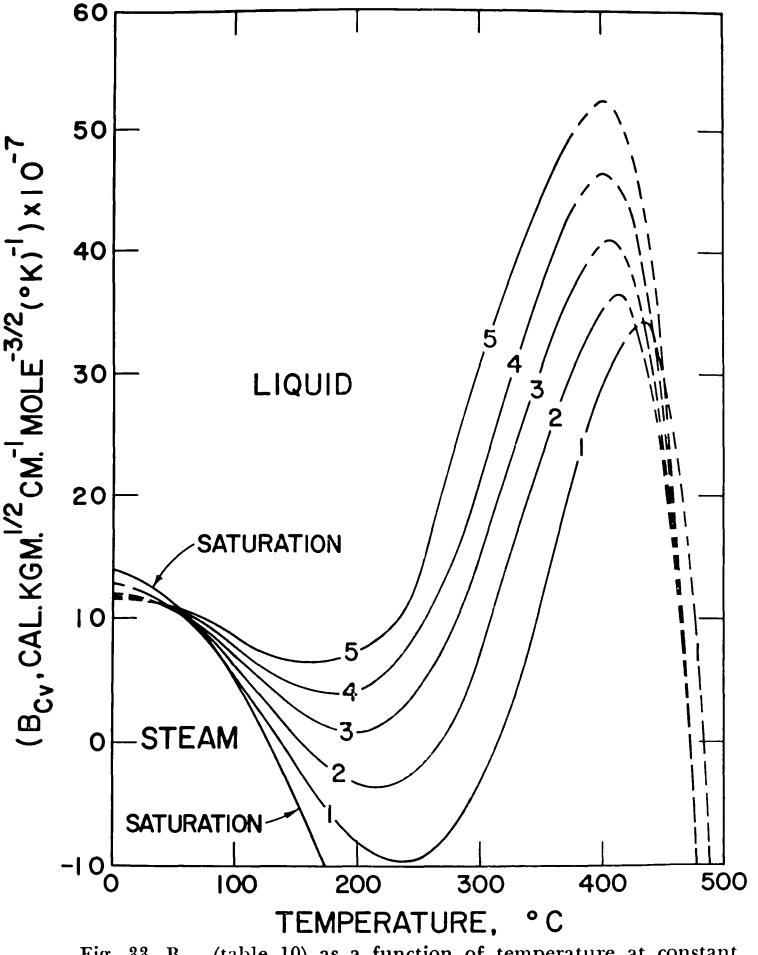


Fig. 33. $B_{cv}$ (table 10) as a function of temperature at constant pressure (labeled in kb) computed from equations (3), (29), (38), and (40).

Values of $A_E$ and $B_E$ computed from equations (26) through (29) are shown in tables 7 and 8 and plotted in figures 22 through 26. $A_E$ as a function of temperature at $\rho = 1$ g cm$^{-3}$ is shown in figure 27, where values computed in this study can be compared with those given by Lewis and Randall (1961), which are based on Franck's (1956) early estimates of the dielectric constant of $H_2O$ at high pressures and temperatures. Franck's dielectric constants were obtained from graphic fits of the Kirkwood equation (Kirkwood, 1939; Oster and Kirkwood, 1943) to relatively low temperature-pressure data. Computer regression (Helgeson and Kirkham, 1974a) of more extensive data including Heger's (ms) recent measurements at high pressures and temperatures yields dielectric constants that differ from Franck's early estimates by as much as 10 percent or more and values of $(\partial \ln \varepsilon/\partial T)_\rho$ that differ by more than 30 percent from corresponding finite difference derivatives calculated by Pitzer and Brewer (Lewis and Randall, 1961) from Franck's data. Because $(\partial \ln \varepsilon/\partial T)_\rho$ is negative, these differences are magnified in equations (28) and (29) by addition of $1/T$, which causes the large departure of the two curves in figure 22.

It can be seen in figures 24 and 26 that the low pressure isobars for $A_E$ and $B_E$ pass through two extrema as functions of temperature, one between 100° and 200°C, and the other in the vicinity of 450°C. The latter extrema also are present in the high pressure isobars, which exhibit increasingly precipitous maxima with increasing pressure. As a consequence of the configuration of the isobars in figures 24 and 26, the isotherms for $A_E$ and $B_E$ in figures 23 and 24 cross each other and exhibit maxima and minima in the steam phase region at 500°C and low pressures. Note that the saturation curves in figures 23 and 24 also maximize. The extrema in $A_E$ and $B_E$, none of which coincides with the critical point (where $(\partial \ln \varepsilon/\partial T)_\rho \neq -\infty = (\partial \ln \varepsilon/\partial T)_P$), are contoured by the isopleths in figures 27 and 28, which describe surfaces somewhat like warped Spanish saddles.

## HEAT CAPACITY AT CONSTANT VOLUME

The relative partial molal heat capacity of the solute at constant volume ($\tilde{C}_{V,2} - \tilde{C}^\circ_{V,2}$) is defined by the isochoric partial derivative of equation (25) with respect to temperature; that is,

$$\tilde{C}_{V,2} - \tilde{C}^\circ_{V,2} = \left(\frac{\partial \tilde{E}_2}{\partial T}\right)_\rho - \left(\frac{\partial \tilde{E}^\circ_2}{\partial T}\right)_\rho$$

$$= -2.303\upsilon R\left(2T\left(\frac{\partial \log \gamma_\pm}{\partial T}\right)_\rho + T^2\left(\frac{\partial^2 \log \gamma_\pm}{\partial T^2}\right)_\rho\right) \quad (36)$$

Similarly, the Debye-Hückel parameters for $\tilde{C}_{V,2} - \tilde{C}^\circ_{V,2}$ (that is, $A_{C_V}$ and

$B_{C_V}$) are given by corresponding derivatives of equations (26) and (27), which appear as

$$A_{C_V} = \left(\frac{\partial A_E}{\partial T}\right)_\rho = 2(2.303)R\left(2T\left(\frac{\partial A_\gamma}{\partial T}\right)_\rho + T^2\left(\frac{\partial^2 A_\gamma}{\partial T^2}\right)_\rho\right)$$
$$= \frac{2A_E}{T} + 2(2.303)RT^2\left(\frac{\partial^2 A_\gamma}{\partial T^2}\right)_\rho \quad (37)$$

and

$$B_{C_V} = \left(\frac{\partial B_E}{\partial T}\right)_\rho = 2(2.303)R\left(2T\left(\frac{\partial B_\gamma}{\partial T}\right)_\rho + T^2\left(\frac{\partial^2 B_\gamma}{\partial T^2}\right)_\rho\right)$$
$$= \frac{2B_E}{T} + 2(2.303)RT^2\left(\frac{\partial^2 B_\gamma}{\partial T^2}\right)_\rho \quad (38)$$

where (from eqs 28 and 29),

$$\left(\frac{\partial^2 A_\gamma}{\partial T^2}\right)_\rho = \frac{1}{A_\gamma}\left(\frac{\partial A_\gamma}{\partial T}\right)_\rho^2 - \frac{3A_\gamma}{2}\left(\left(\frac{\partial^2 \ln \varepsilon}{\partial T^2}\right)_\rho - \frac{1}{T^2}\right) \quad (39)$$

and

$$\left(\frac{\partial^2 B_\gamma}{\partial T^2}\right)_\rho = \frac{1}{B_\gamma}\left(\frac{\partial B_\gamma}{\partial T}\right)_\rho^2 - \frac{B_\gamma}{2}\left(\left(\frac{\partial^2 \ln \varepsilon}{\partial T^2}\right)_\rho - \frac{1}{T^2}\right)$$
$$= \frac{B_\gamma}{3A_\gamma}\left(\frac{\partial^2 A_\gamma}{\partial T^2}\right)_\rho + \frac{1}{3A_\gamma}\left(\frac{\partial B_\gamma}{\partial T}\right)_\rho\left(\frac{\partial A_\gamma}{\partial T}\right)_\rho - \frac{B_\gamma}{3A_\gamma^2}\left(\frac{\partial A_\gamma}{\partial T}\right)_\rho^2 \quad (40)$$

Hence

$$B_{C_V} = \frac{A_E B_E A_\gamma - A_E^2 B_\gamma}{6A_\gamma^2(2.303)RT^2} + \frac{B_\gamma A_{C_V}}{3A_\gamma} \quad (41)$$

The Debye-Hückel equation for $\tilde{C}_{V,2} - \tilde{C}^\circ_{V,2}$ analogous to equation (20) can now be written as

$$\tilde{C}_{V,2} - \tilde{C}^\circ_{V,2} = \frac{\omega A_{C_V} I^{1/2}}{1 + \mathring{a}B_\gamma I^{1/2}} - \frac{\omega I(A_E(\mathring{a}B_E + B_\gamma \mathring{a}_E) + A_\gamma \mathring{a}_E B_E)}{2.303RT^2(1 + \mathring{a}B_\gamma I^{1/2})^2}$$
$$- \frac{\omega A_\gamma I(\mathring{a}B_{C_V} + B_\gamma \mathring{a}_{C_V})}{(1 + \mathring{a}B_\gamma I^{1/2})^2} + \frac{\omega A_\gamma I^{3/2}(\mathring{a}B_E + B_\gamma \mathring{a}_E)^2}{2.303RT^2(1 + \mathring{a}B_\gamma I^{1/2})^3} \quad (42)$$

which reduces in the Guggenheim (1935) modification (eq 4 with $\mathring{a}B_\gamma = 1$) to

$$\tilde{C}_{V,2} - \tilde{C}^\circ_{V,2} = \frac{\omega A_{C_V} I^{1/2}}{1 + I^{1/2}} - \frac{\upsilon b_{C_V} I}{2} \quad (43)$$

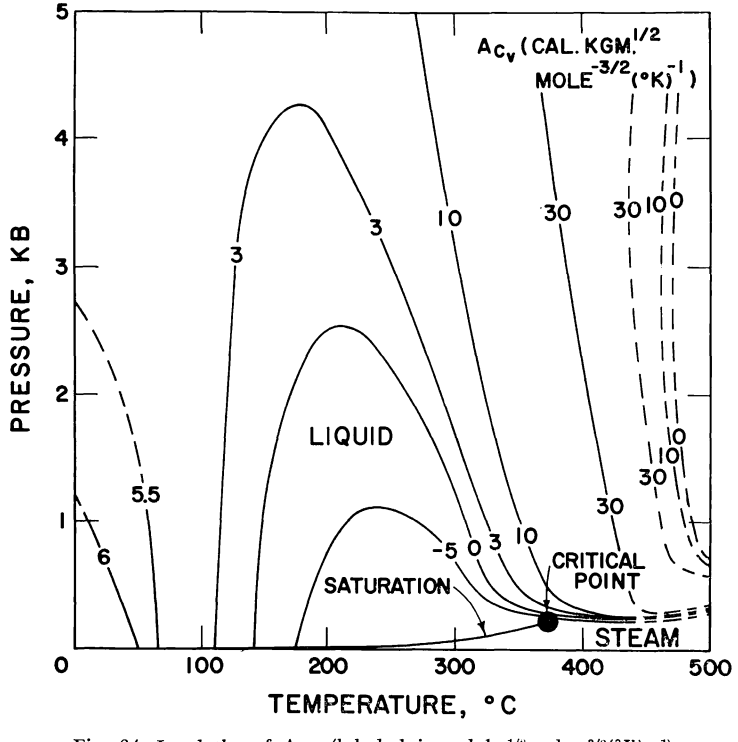


Fig. 34. Isopleths of $A_{C_V}$ (labeled in cal $kg^{1/2}mole^{-3/2}(°K)^{-1}$) as a function of pressure and temperature (table 9 and figs. 30 and 31).

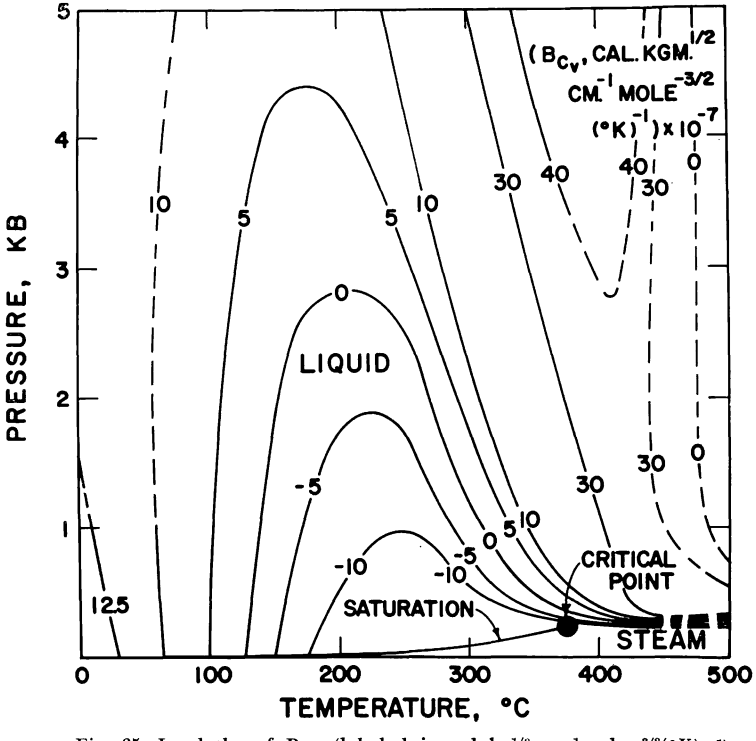


Fig. 35. Isopleths of $B_{C_V}$ (labeled in cal $kg^{1/2}cm^{-1}mole^{-3/2}(°K)^{-1}$) $\times 10^{-7}$) as functions of pressure and temperature (table 10 and figs. 32 and 33).

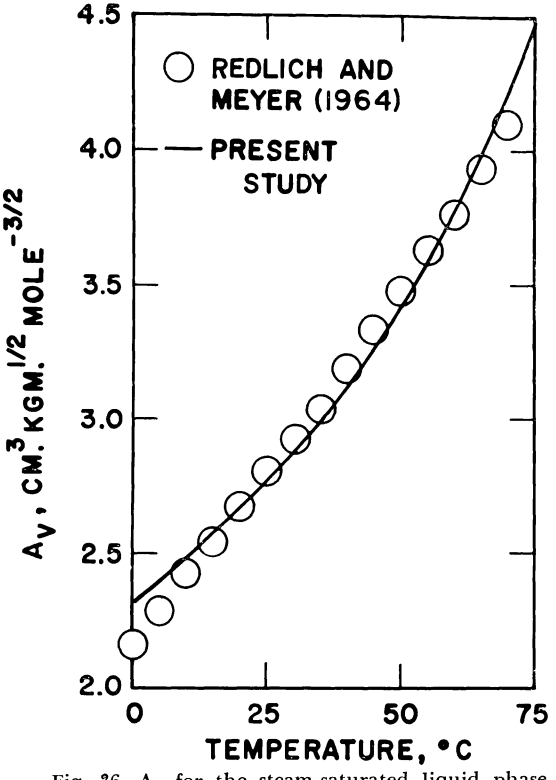


Fig. 36. $A_V$ for the steam-saturated liquid phase as a function of temperature computed (curve) in the present study (table 17) compared with corresponding values taken from the literature.

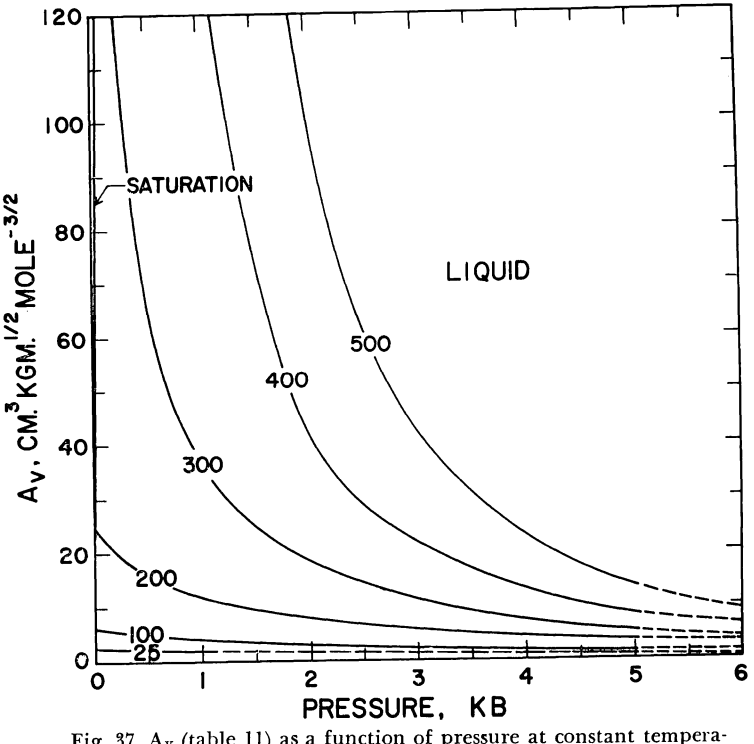


Fig. 37. $A_V$ (table 11) as a function of pressure at constant temperature (labeled in °C) computed from equations (2), (48), and (51).

and thus

$$\phi_{C_V} = \tilde{C}^\circ_{V,2} + \omega A_{C_V} I^{1/2}\left( (1 + I^{1/2})^{-1} - \frac{\sigma(I^{1/2})}{3}\right) - \frac{\upsilon b_{C_V} I}{4} \quad (44)$$

where $\phi_{C_V}$ is the apparent molal isochoric heat capacity of the solute at constant temperature,

$$\begin{aligned} \text{å}_{C_V} = \left(\frac{\partial \text{å}_E}{\partial T}\right)_\rho &= 2(2.303)R\left( 2T\left(\frac{\partial \text{å}}{\partial T}\right)_\rho + T^2\left(\frac{\partial^2 \text{å}}{\partial T^2}\right)_\rho \right) \\ &= \frac{2\text{å}_E}{T} + 2(2.303)RT^2\left(\frac{\partial^2 \text{å}}{\partial T^2}\right)_\rho \end{aligned} \quad (45)$$

and

$$\begin{aligned} b_{C_V} = \left(\frac{\partial b_E}{\partial T}\right)_\rho &= 2(2.303)R\left( 2T\left(\frac{\partial b_\gamma}{\partial T}\right)_\rho + T^2\left(\frac{\partial^2 b_\gamma}{\partial T^2}\right)_\rho \right) \\ &= \frac{2b_E}{T} + 2(2.303)RT^2\left(\frac{\partial^2 b_\gamma}{\partial T^2}\right)_\rho \end{aligned} \quad (46)$$

Equations (43) and (44) are the isochoric analogs of equations (21) and (22).

Values of $A_{C_V}$ and $B_{C_V}$ computed from equations (37) through (40) are shown in tables 9 and 10 and plotted in figures 29 through 33. It can be seen in figure 29 that (as a consequence of the difference in the curves shown in figure 22) the values of $A_{C_V}$ computed in this study for $\rho = 1$ g cm$^3$ differ considerably from those given by Lewis and Randall (1961), which are based on Franck's (1956) early estimates of the dielectric constant at high pressures and temperatures. As indicated above, Franck's values have since been superseded (Heger, ms; Helgeson and Kirkham, 1974a).

The curves in figures 30 through 35 describe smooth high-temperature anticlinal and low-temperature synclinal surfaces, which in the case of $B_{C_V}$ (figs. 32, 33, and 35) and $A_{C_V}$ from $\sim$ 200° to $\sim$ 300°C (figs. 30, 31, and 34) plunge toward lower pressures. The high temperature anticlinal surface for $A_{C_V}$ is doubly plunging toward $\sim$ a kilobar, which causes the angular hook in the 400°C isotherm in figure 30. At temperatures below $\sim$ 50° to 100°C and pressures $\lesssim$ 2 kb, both $A_{C_V}$ and $B_{C_V}$ are negative functions of pressure.

## VOLUME

The partial derivative of equation (1) with respect to pressure at constant temperature can be written as

$$\begin{aligned} \bar{V}_2 - \bar{V}^\circ_2 &= 2.303\upsilon RT\left(\frac{\partial \log \gamma_\pm}{\partial P}\right)_T \\ &= \frac{\omega A_V I^{1/2}}{1 + \text{å}B_\gamma I^{1/2}} + \frac{\omega A_\gamma I(\text{å}B_V + B_\gamma \text{å}_V)}{(1 + \text{å}B_\gamma I^{1/2})^2} \end{aligned} \quad (47)$$

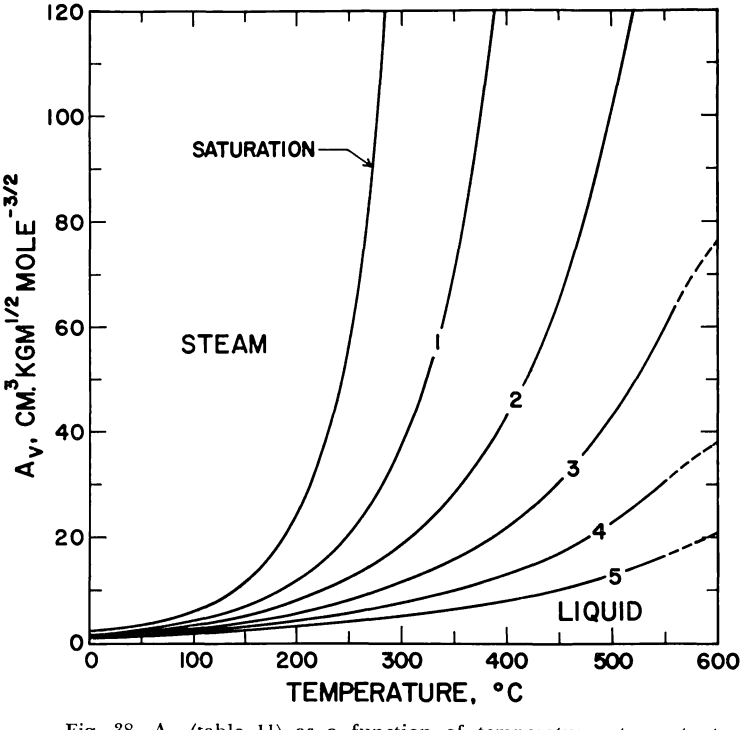


Fig. 38. $A_V$ (table 11) as a function of temperature at constant pressure (labeled in kb) computed from equations (2), (48), and (51).

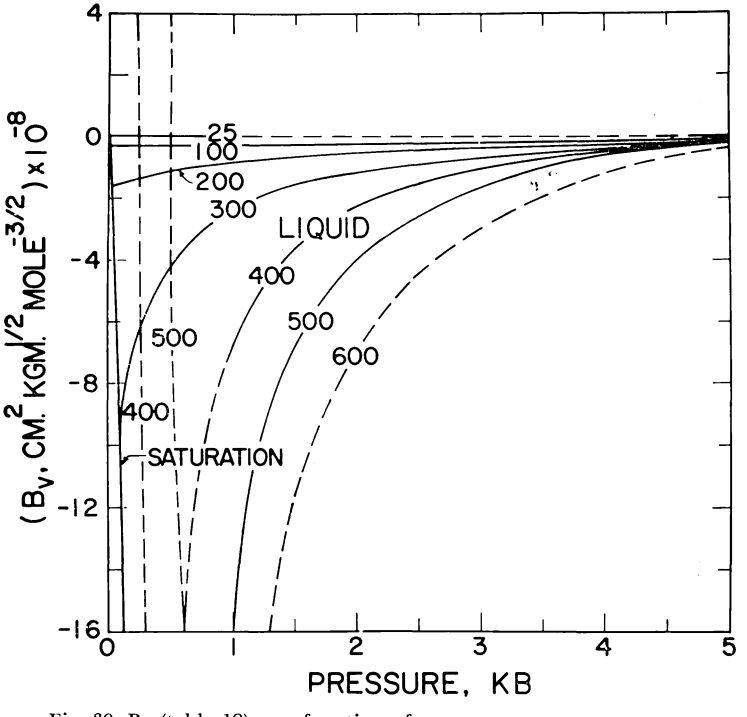


Fig. 39. $B_V$ (table 12) as a function of pressure at constant temperature (labeled in °C) computed from equations (3), (49), and (52).

where $\bar{V}_2 - \bar{V}^\circ_2$ is the relative partial molal volume of the solute at constant pressure and temperature, and $A_V$, $B_V$, and $\mathring{a}_V$ stand for the Debye-Hückel parameters defined by

$$A_V = -2(2.303)RT\left(\frac{\partial A_\gamma}{\partial P}\right)_T \tag{48}$$

$$B_V = 2(2.303)RT\left(\frac{\partial B_\gamma}{\partial P}\right)_T \tag{49}$$

and

$$\mathring{a}_V = 2(2.303)RT\left(\frac{\partial \mathring{a}}{\partial P}\right)_T \tag{50}$$

where (from eqs 2 and 3)

$$\left(\frac{\partial A_\gamma}{\partial P}\right)_T = \frac{A_\gamma}{2}\left(\beta - 3\left(\frac{\partial \ln \varepsilon}{\partial P}\right)_T\right) \tag{51}$$

and

$$\left(\frac{\partial B_\gamma}{\partial P}\right)_T = \frac{B_\gamma}{2}\left(\beta - \left(\frac{\partial \ln \varepsilon}{\partial P}\right)_T\right) \tag{52}$$

where $\beta$ is the coefficient of isothermal compressibility $(-(\partial \ln V/\partial P)_T)$ of $H_2O$. If equation (1) is extended to higher concentrations by addition of a $b_\gamma I$ term (eq 4), a corresponding term $(\upsilon b_V I/2$, where $b_V = 2(2.303)RT(\partial b_\gamma/\partial P)_T)$ appears in equation (47). For electrolytes in which $\mathring{a}B_\gamma$ can be regarded as unity at all pressures, the resulting equation reduces to

$$\bar{V}_2 - \bar{V}^\circ_2 = \frac{\omega A_V I^{1/2}}{1 + I^{1/2}} + \frac{\upsilon b_V I}{2} \tag{53}$$

which is consistent with the Guggenheim (1935) equation (eq 4 with $\mathring{a}B_\gamma$ set to unity) and

$$\phi_V = \bar{V}^\circ_2 + \omega A_V I^{1/2}\left((1 + I^{1/2})^{-1} - \frac{\sigma(I^{1/2})}{3}\right) + \frac{b_V \upsilon I}{4} \tag{54}$$

where $\phi_V$ stands for the apparent molal volume of the solute at constant temperature and pressure. If equation (53) is further simplified by inserting 1 for $(1 + I^{1/2})$ in the denominator of Debye-Hückel term, the expression reduces to

$$\bar{V}_2 - \bar{V}^\circ_2 = \omega A_V I^{1/2} + \frac{\upsilon b_V I}{2} \tag{55}$$

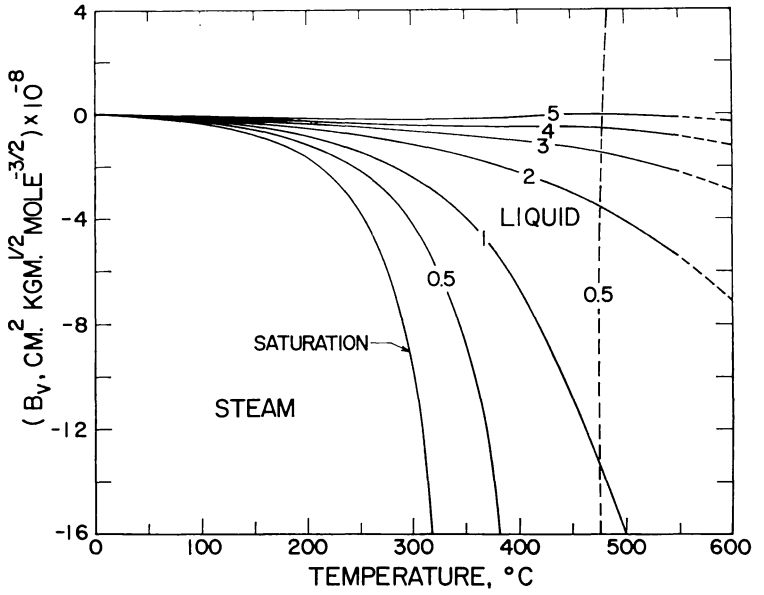


Fig. 40. $B_V$ (table 12) as a function of temperature at constant pressure (labeled in kb) computed from equations (3), (49), and (52).

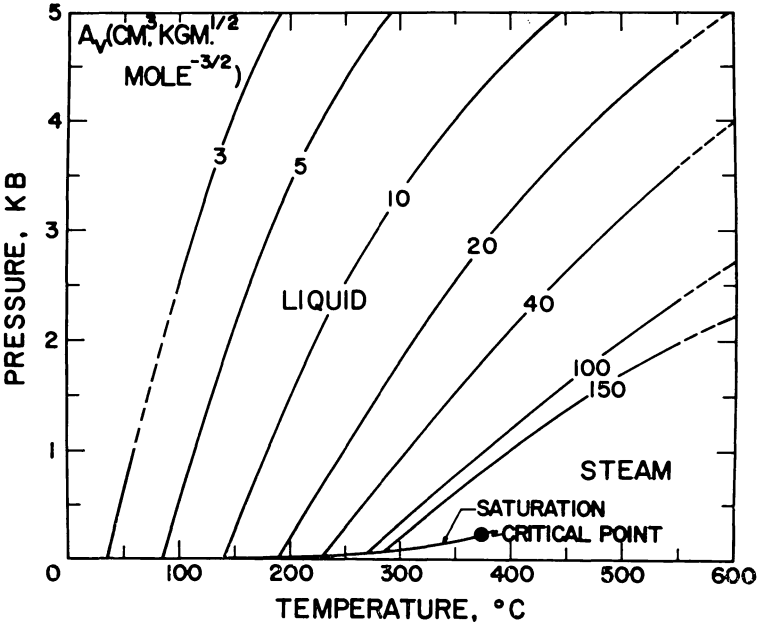


Fig. 41. Isopleths of $A_V$ (labeled in $cm^3kg^{1/2}mole^{-3/2}$) as a function of pressure and temperature (table 11 and figs. 37 and 38).

which is consistent with the Redlich-Meyer (1964) equation for the apparent molal volume; that is,

$$\phi_V = \bar{V}^\circ_2 + S_V m^{1/2} + b^*_V m \tag{56}$$

where

$$S_V = \frac{2A_V\omega^{3/2}}{3} \tag{57}$$

and

$$b^*_V = \frac{\upsilon b_V \omega}{4} \tag{58}$$

If $b_V$ is then set to zero, equation (56) reduces to the molal equivalent of the Masson (1929) equation.

Comparison of $A_V$ values computed in this study with those calculated from Redlich and Meyer's (1964) $k$ values ($A_V = 3/2\ k\rho^{1/2}$) from 0° to 70°C at 1 bar can be made in figure 36, where it can be seen that the two sets of calculations are in close agreement. Nevertheless, the curve in figure 36 departs slightly toward higher values of $A_V$ from those based on Redlich and Meyer's $k$ values as temperature decreases below 20°C and increases above 60°C. Redlich and Meyer computed their $k$ values using Kell and Whalley's (1965) compressibilities (which are in close agreement with compressibilities generated from the equation of state given by Keenan and others, 1969) and values of $(\partial \ln \varepsilon/\partial P)_T$ taken from Owen and others (1961), who represented the dependence of the dielectric constant on temperature and pressure from 0° to 70°C and 1 bar to 1 kb with a quadratic power function. The discrepancy between the distribution of the symbols and the curve in figure 36 arises from differences of ~ 5 percent or less in values of $(\partial \ln \varepsilon/\partial P)_T$ computed by Owen and his coworkers at the edges of their pressure-temperature fit region and those generated by composite regression (Helgeson and Kirkham, 1974a) of data taken from Owen and others (1961), Oshry (ms), and Heger (ms) for temperatures and pressures from 0° to 550°C and 0.001 to 5 kb.

Values of $A_V$ and $B_V$ computed from equations (48), (49), (51), and (52) are shown in tables 11 and 12 and plotted in figures 37 through 40. Comparison of figures 37 and 38 with figures 39 and 40 indicates that (apart from the difference in sign stipulated by eqs 48 and 49) $A_V$ and $B_V$ exhibit much the same dependence on pressure and temperature except in the steam phase region at high temperatures and low pressures.

It can be seen in figure 1 that $\ln \varepsilon$ is nearly a linear function of $\ln V$ at constant temperature, and that the slopes of the isotherms are close to $-1$. As a consequence, $(\partial \ln \varepsilon/dP)_T$ at high temperatures is almost proportional and nearly equivalent in magnitude to $\beta$, both of which are positive and approach $\infty$ at the critical point. It follows from equations (49) and (52) that $B_V$ is a sensitive function of the relative magnitude of $\beta$ and $(\partial \ln \varepsilon/dP)_T$. Except in the steam phase region at high temperatures and low pressures, $(\partial \ln \varepsilon/\partial P)_T > \beta$, which causes $B_V$ to be negative over

most of the temperature/pressure region considered (figs. 39 and 40). However, at high temperatures and low pressures where $\beta > (\partial \ln \varepsilon/dP)_T$, $B_V$ is positive, which causes the low pressure isobars in figure 40 and the high temperature isotherms in figure 39 to pass through precipitous minima at high temperatures and low pressures, respectively. As a consequence, the isopleths for $B_V$ wrap around the critical point in figure 42. The same behavior is not exhibited by $A_V$ in figure 41, because $\beta$ is always smaller than $3(\partial \ln \varepsilon/\partial P)_T$ in equation (51) for the pressure/temperature region considered. Hence $A_V$ decreases monotonically as a function of pressure at constant temperature (fig. 37) and also increases monotonically as an isobaric function of temperature (fig. 38). At high pressures, $A_V \rightarrow B_V \rightarrow (\partial A_V/\partial P)_T \rightarrow (\partial B_V/\partial P)_T \rightarrow 0$ as pressure increases at constant temperature (figs. 37 and 39). Similarly, $A_V \rightarrow B_V \rightarrow (\partial A_V/\partial T)_P \rightarrow (\partial B_V/\partial T)_P \rightarrow 0$ as temperature decreases at constant pressure and low temperatures (figs. 38 and 40).

COMPRESSIBILITY

The relative partial molal compressibility ($\bar{\kappa}_2 - \bar{\kappa}^\circ_2$) of an electrolyte is given by

$$\bar{\kappa} - \bar{\kappa}^\circ_2 = -\left(\left(\frac{\partial \bar{V}_2}{\partial P}\right)_T - \left(\frac{\partial \bar{V}^\circ_2}{\partial P}\right)_T\right) = -2.303\nu RT\left(\frac{\partial^2 \log \gamma_\pm}{\partial P^2}\right)_T \tag{59}$$

Hence, for electrolytes described by equation (1),

$$\bar{\kappa}_2 - \bar{\kappa}^\circ_2 = -\frac{\omega A_\kappa I^{1/2}}{1 + \mathring{a}B_\gamma I^{1/2}} + \frac{\omega I(A_V(\mathring{a}B_V + B_\gamma \mathring{a}_V) + A_\gamma \mathring{a}_V B_V)}{2.303RT\,(1 + \mathring{a}B_\gamma I^{1/2})^2} + \frac{\omega A_\gamma I(\mathring{a}B_\kappa + B_\gamma \mathring{a}_\kappa)}{(1 + \mathring{a}B_\gamma I^{1/2})^2} - \frac{\omega A_\gamma I^{3/2}(\mathring{a}B_V + B_\gamma \mathring{a}_V)^2}{2.303RT\,(1 + \mathring{a}B_\gamma I^{1/2})^3} \tag{60}$$

where

$$A_\kappa = \left(\frac{\partial A_V}{\partial P}\right)_T = -2(2.303)RT\left(\frac{\partial^2 A_\gamma}{\partial P^2}\right)_T, \tag{61}$$

$$B_\kappa = \left(\frac{\partial B_V}{\partial P}\right)_T = 2(2.303)RT\left(\frac{\partial^2 B_\gamma}{\partial P^2}\right)_T, \tag{62}$$

and

$$\mathring{a}_\kappa = \left(\frac{\partial \mathring{a}_V}{\partial P}\right)_T = 2(2.303)RT\left(\frac{\partial^2 \mathring{a}}{\partial P^2}\right)_T \tag{63}$$

where (from eqs 51 and 52)

$$\left(\frac{\partial^2 A_\gamma}{\partial P^2}\right)_T = \frac{1}{A_\gamma}\left(\frac{\partial A_\gamma}{\partial P}\right)^2_T + \frac{A_\gamma}{2}\left(\left(\frac{\partial \beta}{\partial P}\right)_T - 3\left(\frac{\partial^2 \ln \varepsilon}{\partial P^2}\right)_T\right) \tag{64}$$

and

$$\left(\frac{\partial^2 B_\gamma}{\partial P^2}\right)_T = \frac{1}{B_\gamma}\left(\frac{\partial B_\gamma}{\partial P}\right)_T^2 + \frac{B_\gamma}{2}\left(\left(\frac{\partial \beta}{\partial P}\right)_T - \left(\frac{\partial^2 \ln \varepsilon}{\partial P^2}\right)_T\right) \quad (65)$$

Extension of equation (1) to higher concentrations by addition of a $b_\gamma I$ term (eq 4) results in a corresponding extension of equation (60) by $-\upsilon b_\kappa I/2$, where

$$b_\kappa = 2(2.303)RT\left(\frac{\partial b_V}{\partial P}\right)_T \quad (66)$$

The partial derivatives of equations (53) through (56) with respect to pressure at constant temperature can now be written as

$$\bar{\kappa}_2 - \bar{\kappa}^\circ_2 = -\frac{\omega A_\kappa I^{1/2}}{1 + I^{1/2}} - \frac{\upsilon b_\kappa I}{2}, \quad (67)$$

$$\phi_\kappa = \bar{\kappa}^\circ_2 - \omega A_\kappa I^{1/2}\left((1 + I^{1/2})^{-1} - \frac{\sigma(I^{1/2})}{3}\right) - \frac{b_\kappa \upsilon I}{4}, \quad (68)$$

$$\bar{\kappa}_2 - \bar{\kappa}^\circ_2 = -\omega A_\kappa I^{1/2} - \frac{\upsilon b_\kappa I}{2}, \quad (69)$$

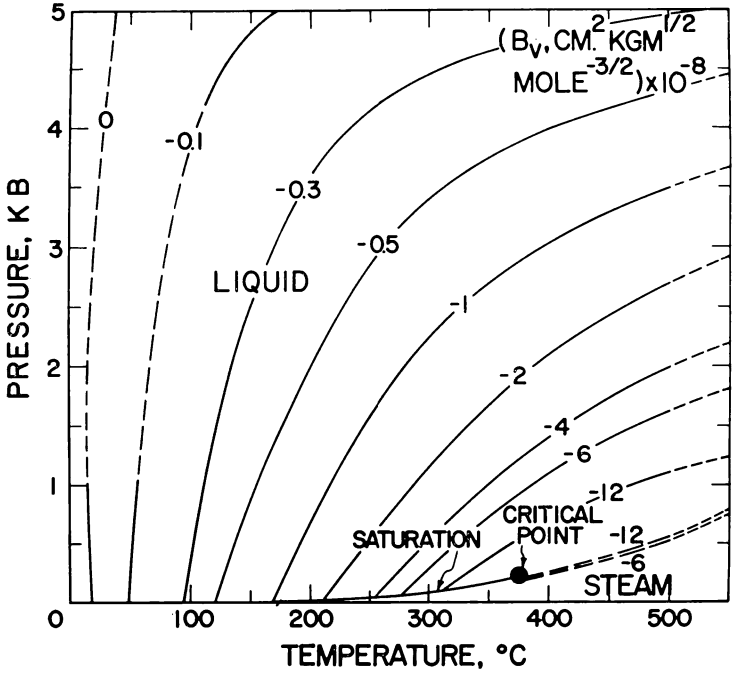


Fig. 42. Isopleths of $B_V$ (labeled in ($cm^2 kg^{1/2} mole^{-3/2}$) $\times 10^{-8}$) as a function of pressure and temperature (table 12 and figs. 39 and 40).

TABLE 13

$A_\kappa$ in $(cm^3 kg^{1/2} mole^{-3/2} bar^{-1}) \times 10^3$ computed from equations (2), (51), (61), and (64)—see figures 43, 44, and 47

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | -1.02 | -0.6 | | | | | |
| 50 | -1.33 | -0.7 | | | | | |
| 75 | -1.91 | -1.1 | | | | | |
| 100 | -2.95 | -1.7 | (-1.1) | (-0.9) | (-0.6) | (-0.4) | (-0.3) |
| 125 | -4.80 | -2.3 | -1.8 | -1.2 | -0.8 | -0.6 | (-0.4) |
| 150 | -8.08 | -4.3 | -2.8 | -1.7 | -1.0 | -0.7 | (-0.5) |
| 175 | -14.13 | -6.8 | -4.3 | -2.3 | -1.4 | -0.9 | (-0.6) |
| 200 | -25.86 | -11.1 | -6.5 | -3.2 | -1.8 | -1.1 | (-0.8) |
| 225 | -50.34 | -18.5 | -9.8 | -4.4 | -2.3 | -1.4 | (-1.0) |
| 250 | -106.98 | -31.7 | -15.1 | -6.1 | -3.0 | -1.7 | (-1.1) |
| 275 | -259.08 | -56.4 | -23.3 | -8.4 | -3.9 | -2.2 | (-1.4) |
| 300 | -772.23 | -106.1 | -36.6 | -11.5 | -5.0 | -2.7 | (-1.6) |
| 325 | -3353.74 | -214.6 | -58.3 | -15.6 | -6.4 | -3.3 | (-2.0) |
| 350 | -36221.30 | -478.9 | -94.4 | -21.1 | -8.1 | -4.0 | (-2.3) |
| 375 | | | -154.7 | -28.5 | -10.2 | -4.8 | (-2.8) |
| 400 | | | -255.8 | -38.6 | -12.9 | -5.8 | (-3.3) |
| 425 | | | -425.3 | -52.3 | -16.3 | -7.1 | (-3.9) |
| 450 | | | ( -706.3) | (-71.0) | (-20.5) | -8.6 | (-4.7) |
| 475 | | | (-1152.9) | (-96.5) | (-25.7) | -10.4 | (-5.5) |
| 500 | | | (-1793.9) | (-130.5) | (-32.3) | (-12.6) | (-6.6) |

TABLE 14

$B_\kappa$ in $(cm^2 kg^{1/2} mole^{-3/2} bar^{-1}) \times 10^{-4}$ computed from equations (3), (52), (62), and (65)—see figures 45, 46, and 48

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | 0.05 | 0.0 | | | | | |
| 50 | 0.31 | 0.2 | | | | | |
| 75 | 0.78 | 0.5 | | | | | |
| 100 | 1.54 | 0.9 | (0.7) | (0.6) | (0.4) | (0.3) | (0.3) |
| 125 | 2.83 | 1.7 | 1.2 | 0.9 | 0.6 | 0.5 | (0.3) |
| 150 | 5.04 | 2.8 | 1.9 | 1.3 | 0.8 | 0.6 | (0.4) |
| 175 | 8.95 | 4.6 | 3.0 | 1.8 | 1.1 | 0.8 | (0.6) |
| 200 | 16.19 | 7.4 | 4.5 | 2.4 | 1.4 | 1.0 | (0.7) |
| 225 | 30.40 | 12.0 | 6.8 | 3.3 | 1.9 | 1.2 | (0.8) |
| 250 | 60.96 | 19.6 | 10.0 | 4.5 | 2.4 | 1.6 | (1.0) |
| 275 | 135.88 | 32.9 | 14.8 | 5.9 | 3.0 | 1.8 | (1.2) |
| 300 | 361.08 | 57.1 | 21.9 | 7.7 | 3.7 | 2.1 | (1.4) |
| 325 | 1330.60 | 104.0 | 32.5 | 10.0 | 4.5 | 2.5 | (1.6) |
| 350 | 10974.44 | 202.3 | 48.1 | 12.7 | 5.4 | 2.9 | (1.8) |
| 375 | | | 70.7 | 16.1 | 6.5 | 3.4 | (2.1) |
| 400 | | | 102.5 | 20.2 | 7.7 | 3.9 | (2.4) |
| 425 | | | 145.0 | 25.3 | 9.2 | 4.5 | (2.7) |
| 450 | | | (197.9) | (31.6) | (10.9) | (5.2) | (3.1) |
| 475 | | | (252.9) | (39.3) | (12.9) | (6.0) | (3.6) |
| 500 | | | | (48.4) | (15.3) | (7.0) | (4.1) |

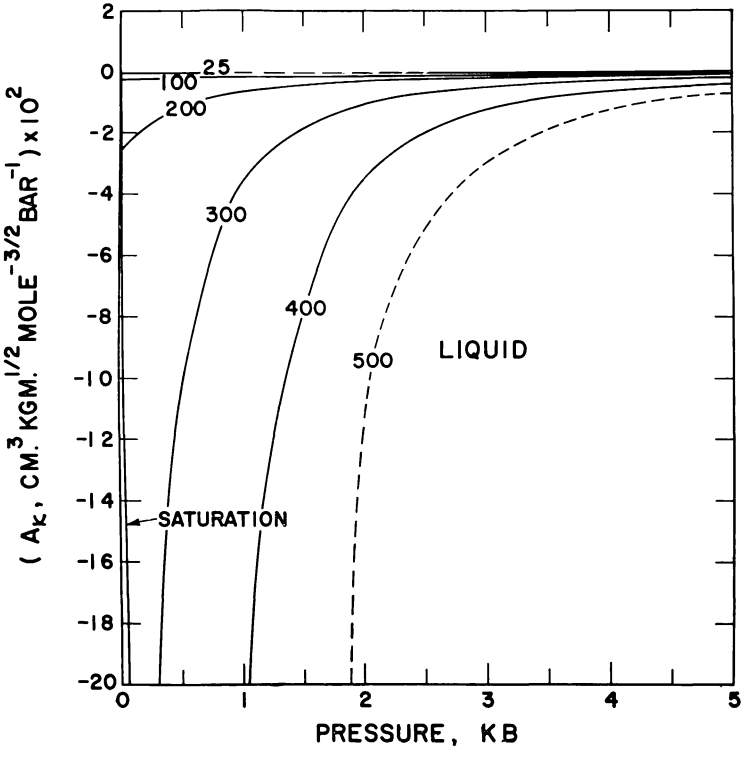


Fig. 43. $A_K$ (table 13) as a function of pressure at constant temperature (labeled in °C) computed from equations (2), (51), (61), and (64).

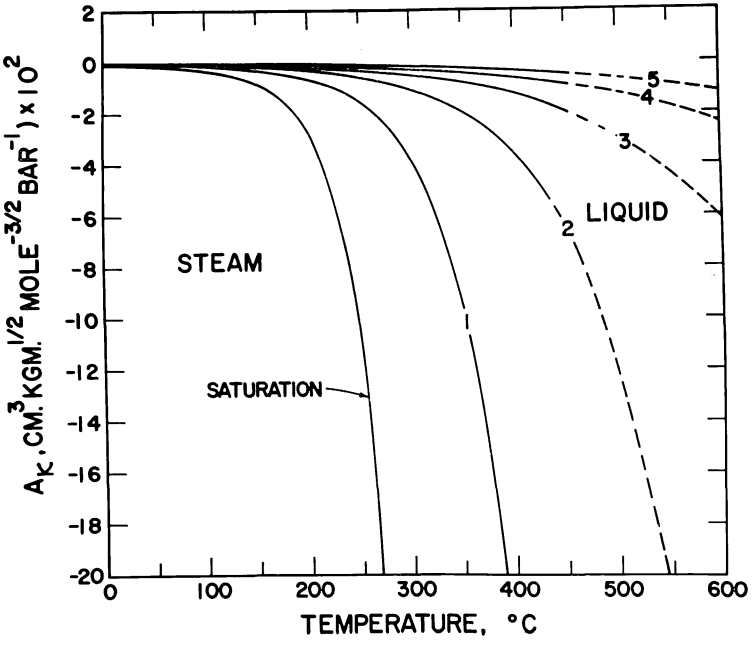


Fig. 44. $A_K$ (table 13) as a function of temperature at constant pressure (labeled in kb) computed from equations (2), (51), (61), and (64).

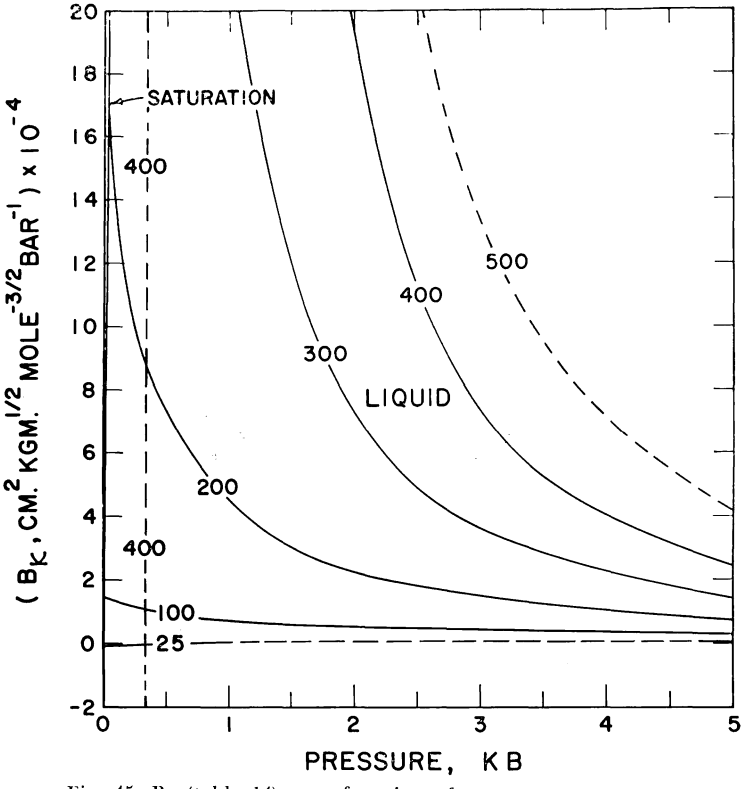


Fig. 45. $B_K$ (table 14) as a function of pressure at constant temperature (labeled in °C) computed from equations (3), (52), (62), and (65).

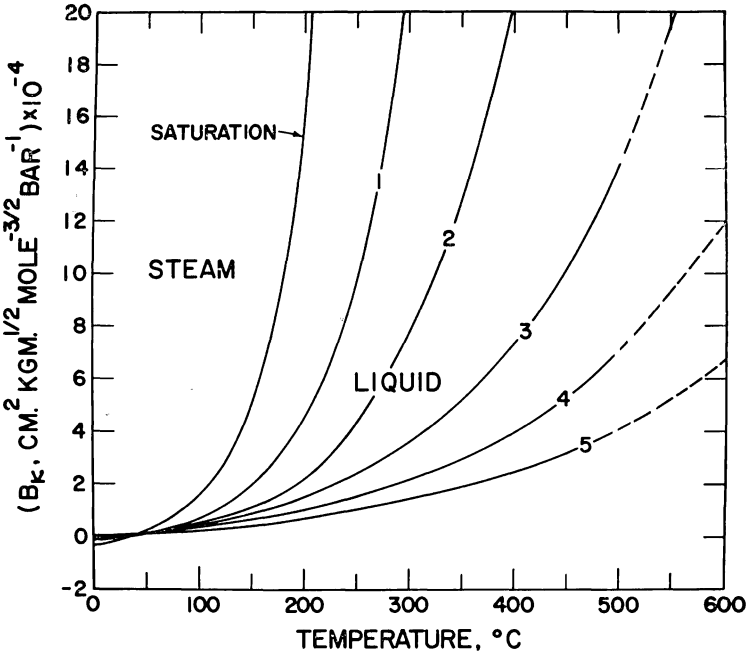


Fig. 46. $B_K$ (table 14) as a function of temperature at constant pressure (labeled in kb) computed from equations (3), (52), (62), and (65).

and

$$\phi_\kappa = \bar{\kappa}^\circ_2 + S_\kappa m^{1/2} + b^*_\kappa m \tag{70}$$

where $\phi_\kappa$ is the apparent molal compressibility of the solute $(-(\partial\phi_V/\partial P)_T$ at constant temperature and pressure,

$$S_\kappa = -\frac{2A_\kappa\omega^{3/2}}{3}\ , \tag{71}$$

and

$$b^*_\kappa = -\frac{\upsilon b_\kappa\omega}{4}\ . \tag{72}$$

With $b_\kappa$ set to zero, equation (70) reduces to the molal equivalent of the Masson (1929) equation for the apparent molal compressibility.

Debye-Hückel compressibility parameters computed from equations (61), (62), (64), and (65) are shown in tables 13 and 14 and plotted in figures 43 through 46. Owing to the near-linear isothermal relation apparent in figure 1, the isothermal dependence of $A_\kappa$ and $B_\kappa$ on pressure (figs. 43 and 45) as well as their isobaric dependence on temperature (figs. 44 and 46) are essentially the same as the corresponding dependence of $-A_V$ and $-B_V$ on pressure and temperature, except at low temperatures.

Because $(\partial A_\gamma/\partial P)_T$ is squared in equation (64) and $(3|(\partial^2 \ln \varepsilon/\partial P^2)_T|) > |(\partial\beta/\partial P)_T|$, both of which are negative, $A_\kappa$ is negative throughout the pressure-temperature region considered, and it becomes more negative with increasing temperature and decreasing pressure (figs. 43 and 44). Except at low temperatures and in the steam phase region at high temperatures and low pressures where $(\partial \ln \varepsilon/\partial P)_T < \beta$ and $|(\partial^2 \ln \varepsilon/\partial P^2)_T| < |(\partial B/\partial P)_T|$, $B_\kappa$ is positive and becomes more positive with increasing temperature and decreasing pressure. At low temperatures ($\lesssim 25°C$ at 1 bar) $B_\kappa$ is negative, but it becomes less so with increasing pressure and temperature. The precipitous minimum in $B_V$ at high temperatures and low pressures is also exhibited by $B_\kappa$ in the steam phase region (for much the same reason), which causes isopleths for $B_\kappa$ at high temperatures and low pressures to curve around the critical point (not shown in fig. 48).

Because of the relations shown in figure 1, the configurations of the isopleths in figures 41 and 47, and 42 and 48, respectively, are similar, except at low temperatures where the isobars in figure 44 maximize and those in figure 46 cross each other as a consequence of the minima exhibited by $\beta$ and $(\partial \ln \varepsilon/\partial P)_T$ as isobaric functions of temperature at low pressures and temperatures. Accordingly, the isopleths for $A_\kappa$ in figure 47 pass through minima at low temperatures and pressures $\lesssim 2$ kb, and those for $B_\kappa$ in figure 48 have negative slopes below $\sim 35°C$. At high pressures $A_\kappa \to (\partial A_\kappa/\partial P)_T \to 0$.

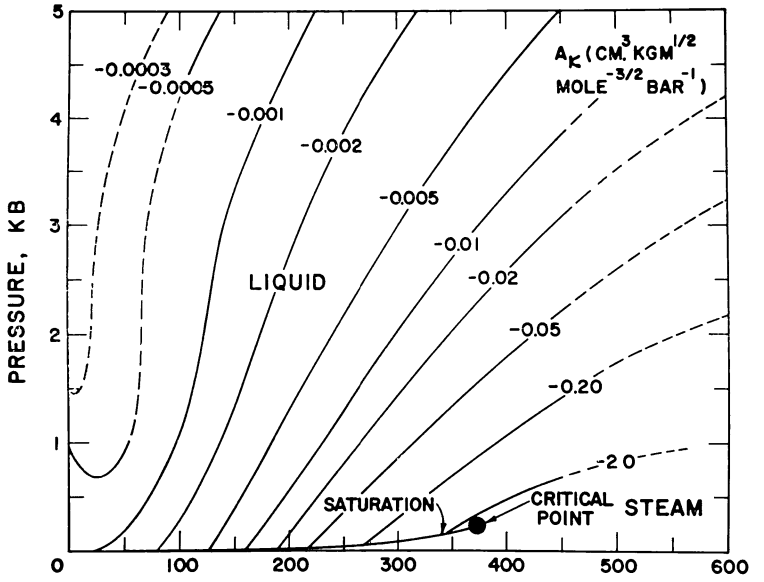


Fig. 47. Isopleths of $A_\kappa$ (labeled in $cm^3kg^{1/2}mole^{-3/2}bar^{-1}$) as a function of pressure and temperature (table 13 and figs. 43 and 44). Temperature, °C.

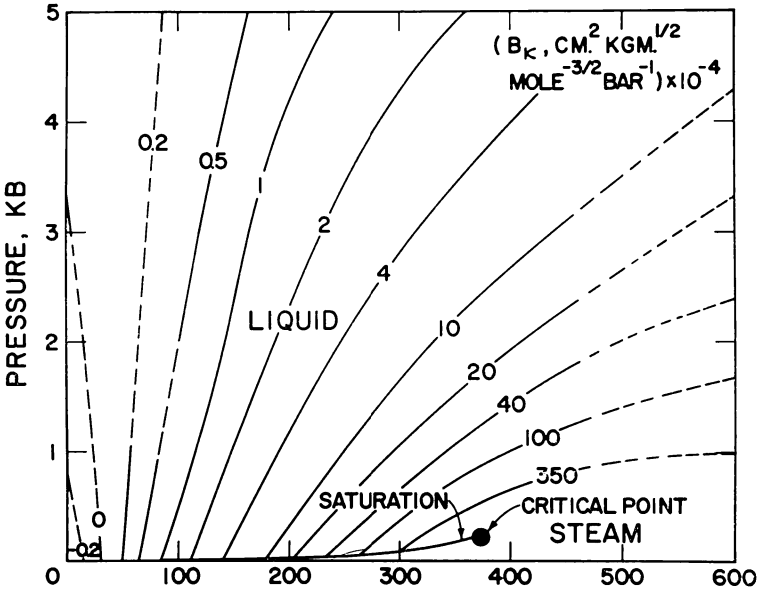


Fig. 48. Isopleths of $B_\kappa$ (labeled in $(cm^2kg^{1/2}mole^{-3/2}bar^{-1}) \times 10^{-4}$) as a function of pressure and temperature (table 14 and figs. 45 and 46). Temperature, °C.

EXPANSIBILITY

The relative partial molal expansibility of an electrolyte at constant pressure and temperature ($\bar{E}x_2 - \bar{E}x^\circ_2$) can be expressed as

$$\bar{E}x_2 - \bar{E}x^\circ_2 = \left(\frac{\partial \bar{V}_2}{\partial T}\right)_P - \left(\frac{\partial \bar{V}^\circ_2}{\partial T}\right)_P$$

$$= 2.303R\left(\left(\frac{\partial \log \gamma_\pm}{\partial T}\right)_P + T\left(\frac{\partial^2 \log \gamma_\pm}{\partial T^2}\right)_P\right)$$

$$= \frac{\bar{V}_2 - \bar{V}^\circ_2}{T} + 2.303RT\left(\frac{\partial^2 \log \gamma_\pm}{\partial T^2}\right)_P \tag{73}$$

which can be combined with the partial derivative of equation (47) to give

$$\bar{E}x_2 - \bar{E}x^\circ_2 = \frac{\omega A_{Ex} I^{1/2}}{1 + \mathring{a}B_\gamma I^{1/2}} + \frac{\omega I(A_H(\mathring{a}B_V + B_\gamma \mathring{a}_V) + A_\gamma(\mathring{a}_H B_V + B_H \mathring{a}_V) - A_V(\mathring{a}B_H + \mathring{a}_H B_\gamma))}{2(2.303)RT^2(1 + \mathring{a}B_\gamma I^{1/2})^2} + \frac{\omega A_\gamma I(\mathring{a}B_{Ex} + B_\gamma \mathring{a}_{Ex})}{(1 + \mathring{a}B_\gamma I^{1/2})^2} - \frac{\omega A_\gamma I^{3/2}(\mathring{a}B_V + B_\gamma \mathring{a}_V)(\mathring{a}_H B_\gamma + \mathring{a}B_H)}{2.303RT^2(1 + \mathring{a}B_\gamma I^{1/2})^3} \tag{74}$$

where

$$A_{Ex} = \left(\frac{\partial A_V}{\partial T}\right)_P = -2(2.303)R\left(\left(\frac{\partial A_\gamma}{\partial P}\right)_T + T\left(\frac{\partial\left(\frac{\partial A_\gamma}{\partial P}\right)_T}{\partial T}\right)_P\right) = \frac{A_V}{T} - \frac{1}{T}\left(\frac{\partial A_H}{\partial P}\right)_T, \tag{75}$$

$$B_{Ex} = \left(\frac{\partial B_V}{\partial T}\right)_P = 2(2.303)R\left(\left(\frac{\partial B_\gamma}{\partial P}\right)_T + T\left(\frac{\partial\left(\frac{\partial B_\gamma}{\partial P}\right)_T}{\partial T}\right)_P\right) = \frac{B_V}{T} + \frac{1}{T}\left(\frac{\partial B_H}{\partial P}\right)_T \tag{76}$$

and

$$\mathring{a}_{Ex} = \left(\frac{\partial \mathring{a}_V}{\partial T}\right)_P = 2(2.303)R\left(\left(\frac{\partial \mathring{a}}{\partial P}\right)_T + T\left(\frac{\partial\left(\frac{\partial \mathring{a}}{\partial P}\right)_T}{\partial T}\right)_P\right) = \frac{\mathring{a}_V}{T} + \frac{1}{T}\left(\frac{\partial \mathring{a}_H}{\partial P}\right)_T \tag{77}$$

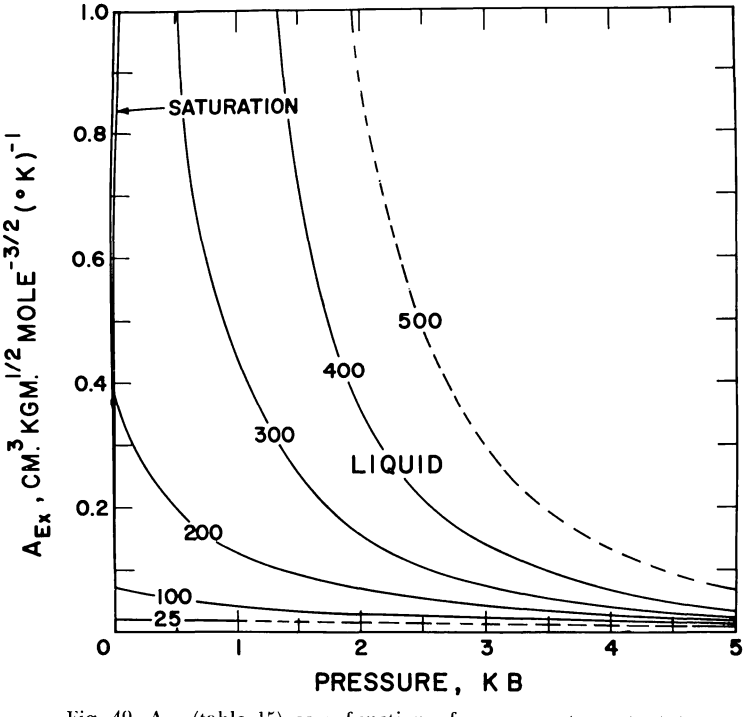


Fig. 49. $A_{Ex}$ (table 15) as a function of pressure at constant temperature (labeled in °C) computed from equations (2), (5), (51), (75), and (78).

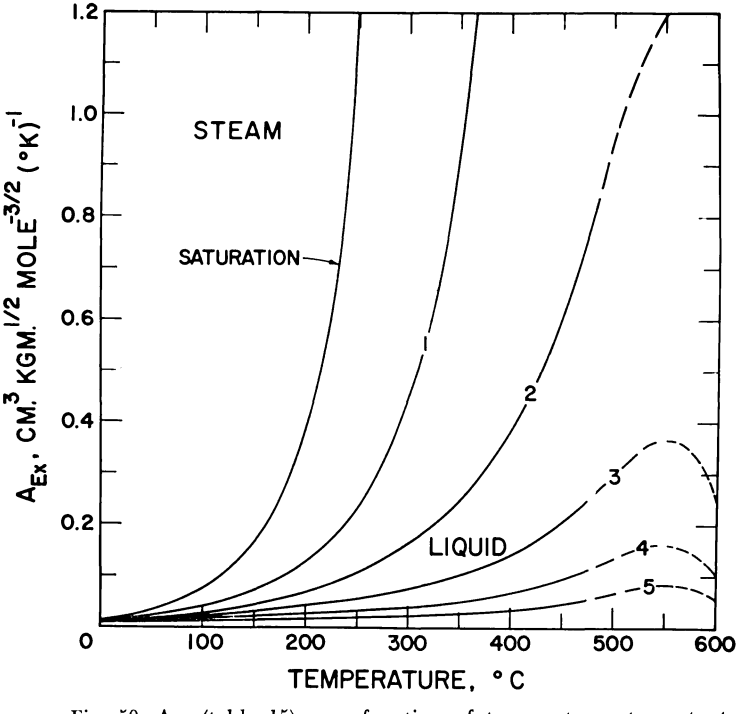


Fig. 50. $A_{Ex}$ (table 15) as a function of temperature at constant pressure (labeled in kb) computed from equations (2), (5), (51), (75), and (78).

TABLE 15

$A_{Ex}$ in $(cm^3 kg^{1/2} mole^{-3/2} (°K)^{-1} \times 10^2$ computed from equations (2), (5), (51), (75), and (78)—see figures 49, 50, and 53

| t | PRESSURE, KB | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 1.5 | 2 | 3 | 4 | 5 |
| 25 | 2.08 | 1.8 | | | | | | |
| 50 | 3.40 | 2.8 | | | | | | |
| 75 | 5.10 | 3.9 | | | | | | |
| 100 | 7.45 | 5.4 | (4.1) | (3.5) | (2.8) | (2.0) | (1.6) | (1.2) |
| 125 | 10.89 | 7.5 | 5.5 | 4.4 | 3.6 | 2.5 | 1.8 | (1.4) |
| 150 | 16.14 | 10.3 | 7.2 | 5.7 | 4.5 | 3.0 | 2.1 | (1.6) |
| 175 | 24.58 | 14.4 | 9.5 | 7.4 | 5.6 | 3.5 | 2.4 | (1.8) |
| 200 | 39.02 | 20.4 | 12.6 | 9.5 | 7.0 | 4.1 | 2.7 | (1.9) |
| 225 | 65.64 | 29.6 | 17.0 | 12.2 | 8.7 | 4.8 | 3.0 | (2.1) |
| 250 | 119.90 | 44.5 | 23.1 | 15.5 | 10.7 | 5.6 | 3.4 | (2.2) |
| 275 | 246.93 | 69.7 | 32.0 | 19.7 | 13.1 | 6.5 | 3.7 | (2.3) |
| 300 | 614.27 | 114.9 | 45.2 | 25.0 | 16.0 | 7.4 | 4.1 | (2.4) |
| 325 | 2160.46 | 201.2 | 65.0 | 32.0 | 19.6 | 8.6 | 4.5 | (2.6) |
| 350 | 17927.28 | 381.4 | 94.8 | 41.3 | 24.1 | 10.0 | 5.0 | (2.8) |
| 375 | | | 139.2 | 54.0 | 29.9 | 11.8 | 5.7 | (3.1) |
| 400 | | | 204.3 | 71.8 | 37.5 | 14.1 | 6.6 | (3.4) |
| 425 | | | 298.8 | 96.4 | 47.6 | 17.1 | 7.8 | (4.0) |
| 450 | | | (433.2) | (129.9) | (60.7) | (20.7) | (9.3) | (4.7) |
| 475 | | | (613.2) | (173.2) | (77.5) | (25.1) | (11.1) | (5.5) |
| 500 | | | (819.0) | (224.8) | (97.7) | (29.9) | (13.0) | (6.5) |

TABLE 16

$B_{Ex}$ in $(cm^2 kg^{1/2} mole^{-3/2} (°K)^{-1}) \times 10^{-5}$ computed from equations (3), (6), (52), (76), and (79)—see figures 51, 52, and 54

| t | PRESSURE, KB | | | | | | |
|---|---|---|---|---|---|---|---|
| (°C) | SAT | 0.5 | 1 | 2 | 3 | 4 | 5 |
| 25 | -3.12 | -2.6 | | | | | |
| 50 | -3.58 | -3.0 | | | | | |
| 75 | -4.63 | -3.7 | | | | | |
| 100 | -6.24 | -4.8 | (-3.8) | (-2.7) | (-1.9) | (-1.4) | (-1.0) |
| 125 | -8.58 | -6.2 | -4.7 | -3.1 | -2.2 | -1.6 | (-1.1) |
| 150 | -12.01 | -8.0 | -5.8 | -3.7 | -2.4 | -1.6 | (-1.1) |
| 175 | -17.23 | -10.6 | -7.2 | -4.3 | -2.6 | -1.7 | (-1.1) |
| 200 | -25.59 | -14.1 | -9.0 | -5.0 | -2.8 | -1.7 | (-1.0) |
| 225 | -39.99 | -19.1 | -11.3 | -5.8 | -3.0 | -1.6 | (-0.8) |
| 250 | -67.13 | -26.7 | -14.3 | -6.6 | -3.1 | -1.5 | (-0.6) |
| 275 | -124.99 | -38.5 | -18.4 | -7.4 | -3.1 | -1.3 | (-0.2) |
| 300 | -274.07 | -57.7 | -24.0 | -8.2 | -3.2 | -1.1 | ( 0.0) |
| 325 | -814.22 | -89.8 | -31.6 | -9.2 | -3.2 | -0.8 | (-0.4) |
| 350 | -5197.56 | -146.5 | -41.7 | -10.5 | -3.3 | -0.6 | (-0.8) |
| 375 | | | -54.2 | -11.7 | -3.5 | -0.4 | (-1.1) |
| 400 | | | -69.1 | -13.6 | -3.9 | -0.4 | (-1.3) |
| 425 | | | -84.7 | -16.2 | -4.6 | -0.6 | (-1.4) |
| 450 | | | (-99.1) | (-19.6) | (-5.8) | (-1.0) | (-1.2) |
| 475 | | | (-107.1) | (-24.2) | (-7.4) | (-1.9) | (-0.8) |
| 500 | | | (-99.2) | (-29.7) | (-9.6) | (-3.0) | ( 0.0) |

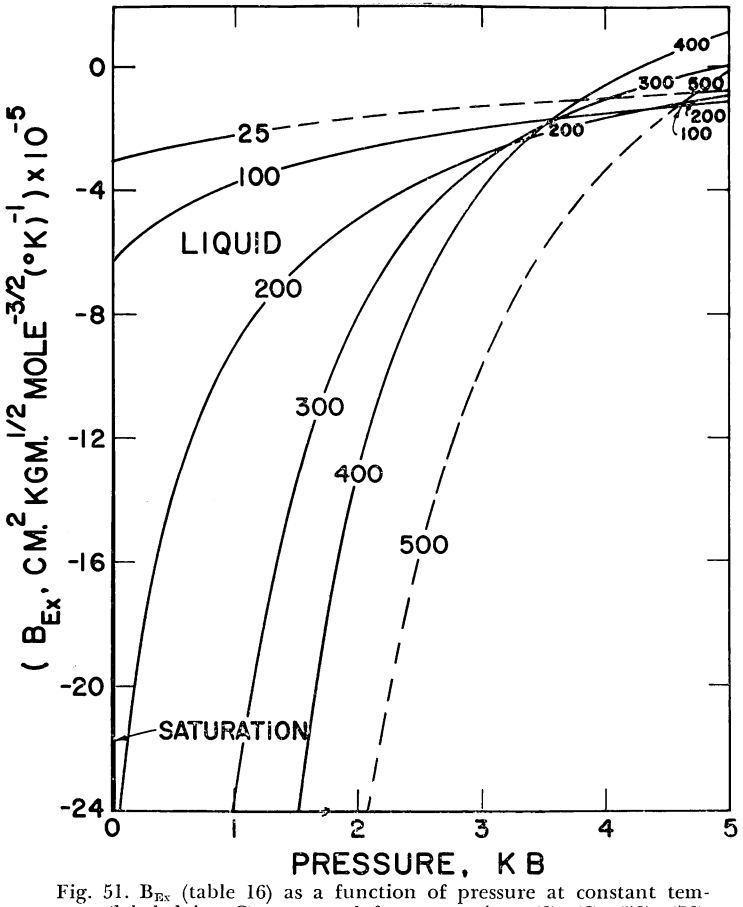


Fig. 51. $B_{Ex}$ (table 16) as a function of pressure at constant temperature (labeled in °C) computed from equations (3), (6), (52), (76), and (79).

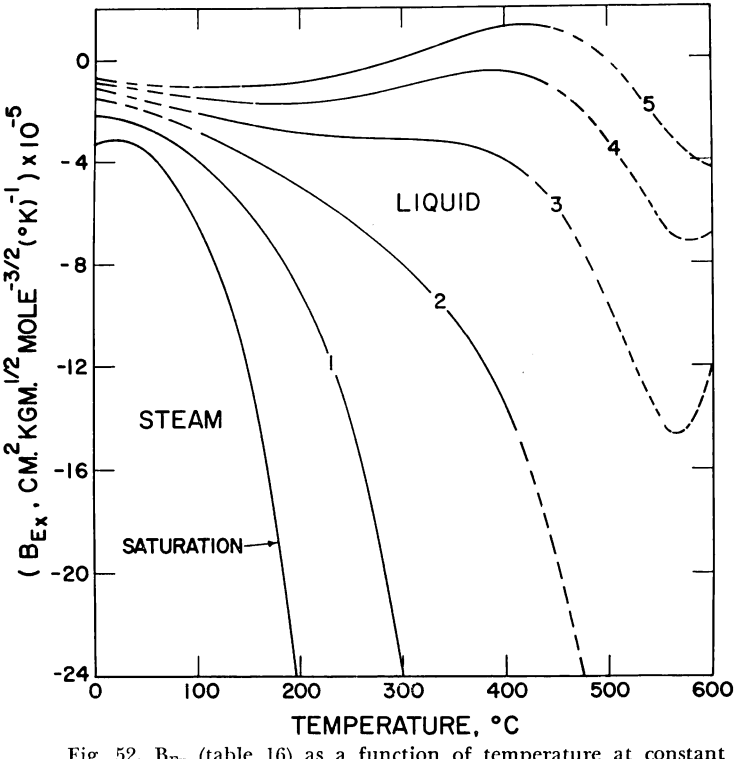


Fig. 52. $B_{Ex}$ (table 16) as a function of temperature at constant pressure (labeled in kb) computed from equations (3), (6), (52), (76), and (79).

where (from eqs 51 and 52)

$$\left(\frac{\partial\left(\frac{\partial A_\gamma}{\partial P}\right)_T}{\partial T}\right)_P = \frac{1}{A_\gamma}\left(\frac{\partial A_\gamma}{\partial P}\right)_T\left(\frac{\partial A_\gamma}{\partial T}\right)_P + \frac{A_\gamma}{2}\left(\left(\frac{\partial \beta}{\partial T}\right)_P - 3\left(\frac{\partial\left(\frac{\partial \ln \varepsilon}{\partial P}\right)_T}{\partial T}\right)_P\right) = -\frac{A_V A_H}{4A_\gamma (2.303R)^2 T^3} + \frac{A_\gamma}{2}\left(\left(\frac{\partial \beta}{\partial T}\right)_P - 3\left(\frac{\partial\left(\frac{\partial \ln \varepsilon}{\partial P}\right)_T}{\partial T}\right)_P\right) \quad (78)$$

and

$$\left(\frac{\partial\left(\frac{\partial B_\gamma}{\partial P}\right)_T}{\partial T}\right)_P = \frac{1}{B_\gamma}\left(\frac{\partial B_\gamma}{\partial P}\right)_T\left(\frac{\partial B_\gamma}{\partial T}\right)_P + \frac{B_\gamma}{2}\left(\left(\frac{\partial \beta}{\partial T}\right)_P - \left(\frac{\partial\left(\frac{\partial \ln \varepsilon}{\partial P}\right)_T}{\partial T}\right)_P\right) = \frac{B_V B_H}{4A_\gamma (2.303R)^2 T^3} + \frac{A_\gamma}{2}\left(\left(\frac{\partial \beta}{\partial T}\right)_P - \left(\frac{\partial\left(\frac{\partial \ln \varepsilon}{\partial P}\right)_T}{\partial T}\right)_P\right) \quad (79)$$

Addition of a $b_\gamma I$ term to the right side of equation (1) to produce equation (4) results in a corresponding term ($\upsilon b_{Ex} I/2$) in equation (74), in which

$$b_{Ex} = 2(2.303)RT\left(\frac{\partial b_V}{\partial T}\right)_P = 2(2.303)R\left(\left(\frac{\partial b_\gamma}{\partial P}\right)_T + T\left(\frac{\partial\left(\frac{\partial b_\gamma}{\partial P}\right)_T}{\partial T}\right)_P\right) = \frac{b_V}{T} + \frac{1}{T}\left(\frac{\partial b_H}{\partial P}\right)_T \quad (80)$$

The expansibility analogs of equations (67) through (70) can now be written as

$$\bar{E}x_2 - \bar{E}x^\circ_2 = \frac{\omega A_{Ex} I^{1/2}}{1 + I^{1/2}} + \frac{\upsilon b_{Ex} I}{2}, \quad (81)$$

$$\phi_{Ex} = \bar{E}x^\circ_2 + \omega A_{Ex} I^{1/2}\left((1 + I^{1/2})^{-1} - \frac{\sigma(I^{1/2})}{3}\right) + \frac{b_{Ex}\upsilon I}{4}, \quad (82)$$

$$\bar{E}x_2 - \bar{E}x^\circ_2 = \omega A_{Ex} I^{1/2} + \frac{\upsilon b_{Ex} I}{2}, \quad (83)$$

and

$$\phi_{Ex} = \bar{E}x^{\circ}_{2} + S_{Ex}\, m^{1/2} + b^{*}_{Ex} m \quad , \tag{84}$$

where $\phi_{Ex}$ is the apparent molal expansibility of the solute at constant temperature and pressure $((\partial\phi_V/\partial T)_P)$,

$$S_{Ex} = \frac{2A_{Ex}\omega^{3/2}}{3} \quad , \tag{85}$$

and

$$b^{*}_{Ex} = \frac{\upsilon b_{Ex}\omega}{4} \tag{86}$$

Setting $b_{Ex} = 0$ reduces equation (84) to the molal analog of the Masson (1929) equation for expansibility.

Values of $A_{Ex}$ and $B_{Ex}$ computed from equations (75), (76), (78), and (79) are shown in tables 15 and 16 and plotted in figures 49 through 52. Owing to the monotonic and negative isothermal dependence of $A_V$ and $A_H$ (both of which are positive) on pressure in figures 9 and 37, $A_{Ex}$ is positive but exhibits a negative dependence on pressure (fig. 49). Similarly, as a consequence of the maxima in the isotherms in figure 10, $A_{Ex}$ passes through an extremum as a function of temperature at constant

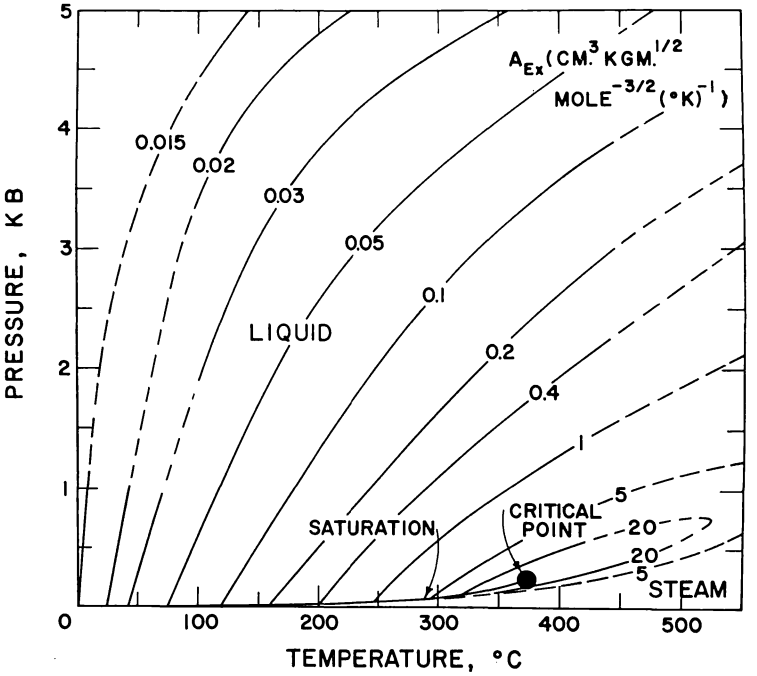


Fig. 53. Isopleths of $A_{Ex}$ (labeled in $cm^3 kg^{1/2} mole^{-3/2}(^{\circ}K)^{-1}$) as a function of pressure and temperature (table 15 and figs. 49 and 50).

TABLE 17

Summary of molal Debye-Hückel A (limiting law) parameters for the steam-saturated liquid phase

| t[a] | P[b] | $A_\gamma$[c] | $A_H$[d] | $A_j$[e] | $A_E$[d] | $A_{C_V}$[e] | $A_V$[f] | $A_K \times 10^3$[g] | $A_{Ex} \times 10^2$[h] |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.006 | 0.4913 | 0.392 | 10.27 | 0.408 | 6.67 | 2.33 | -0.56 | 1.49 |
| 5 | 0.009 | 0.4943 | 0.445 | 11.18 | 0.441 | 6.67 | 2.41 | -0.73 | 1.43 |
| 10 | 0.012 | 0.4976 | 0.503 | 11.84 | 0.475 | 6.67 | 2.48 | -0.84 | 1.50 |
| 15 | 0.017 | 0.5012 | 0.563 | 12.36 | 0.510 | 6.66 | 2.56 | -0.91 | 1.65 |
| 20 | 0.023 | 0.5050 | 0.626 | 12.80 | 0.545 | 6.64 | 2.65 | -0.97 | 1.85 |
| 25 | 0.032 | 0.5092 | 0.691 | 13.22 | 0.582 | 6.61 | 2.75 | -1.02 | 2.08 |
| 30 | 0.042 | 0.5135 | 0.759 | 13.63 | 0.619 | 6.56 | 2.86 | -1.07 | 2.32 |
| 35 | 0.056 | 0.5182 | 0.828 | 14.05 | 0.657 | 6.49 | 2.98 | -1.12 | 2.57 |
| 40 | 0.074 | 0.5231 | 0.899 | 14.49 | 0.695 | 6.41 | 3.12 | -1.18 | 2.84 |
| 45 | 0.096 | 0.5282 | 0.973 | 14.95 | 0.734 | 6.32 | 3.26 | -1.25 | 3.11 |
| 50 | 0.123 | 0.5336 | 1.049 | 15.44 | 0.773 | 6.20 | 3.43 | -1.33 | 3.40 |
| 55 | 0.158 | 0.5392 | 1.127 | 15.97 | 0.812 | 6.06 | 3.60 | -1.42 | 3.70 |
| 60 | 0.199 | 0.5450 | 1.208 | 16.53 | 0.852 | 5.90 | 3.80 | -1.52 | 4.02 |
| 65 | 0.250 | 0.5511 | 1.292 | 17.13 | 0.892 | 5.72 | 4.01 | -1.63 | 4.36 |
| 70 | 0.312 | 0.5574 | 1.380 | 17.76 | 0.932 | 5.52 | 4.23 | -1.76 | 4.72 |
| 75 | 0.386 | 0.5639 | 1.470 | 18.45 | 0.972 | 5.29 | 4.48 | -1.91 | 5.10 |
| 80 | 0.474 | 0.5706 | 1.564 | 19.17 | 1.011 | 5.03 | 4.74 | -2.07 | 5.51 |
| 85 | 0.578 | 0.5776 | 1.662 | 19.95 | 1.051 | 4.75 | 5.03 | -2.26 | 5.94 |
| 90 | 0.701 | 0.5848 | 1.764 | 20.77 | 1.090 | 4.45 | 5.34 | -2.46 | 6.41 |
| 95 | 0.845 | 0.5922 | 1.870 | 21.64 | 1.129 | 4.11 | 5.67 | -2.70 | 6.91 |
| 100 | 1.013 | 0.5998 | 1.980 | 22.57 | 1.167 | 3.75 | 6.03 | -2.95 | 7.45 |
| 105 | 1.208 | 0.6077 | 2.095 | 23.56 | 1.204 | 3.36 | 6.41 | -3.25 | 8.03 |
| 110 | 1.433 | 0.6158 | 2.215 | 24.61 | 1.241 | 2.94 | 6.83 | -3.57 | 8.66 |
| 115 | 1.691 | 0.6242 | 2.341 | 25.73 | 1.277 | 2.50 | 7.28 | -3.93 | 9.35 |
| 120 | 1.985 | 0.6328 | 2.472 | 26.91 | 1.312 | 2.02 | 7.76 | -4.34 | 10.08 |
| 125 | 2.321 | 0.6416 | 2.610 | 28.17 | 1.346 | 1.52 | 8.29 | -4.80 | 10.89 |
| 130 | 2.701 | 0.6507 | 2.754 | 29.52 | 1.378 | 0.98 | 8.85 | -5.31 | 11.76 |
| 135 | 3.130 | 0.6601 | 2.904 | 30.96 | 1.409 | 0.42 | 9.46 | -5.89 | 12.71 |
| 140 | 3.613 | 0.6697 | 3.063 | 32.50 | 1.439 | -0.16 | 10.12 | -6.53 | 13.75 |
| 145 | 4.154 | 0.6796 | 3.228 | 34.15 | 1.467 | -0.78 | 10.83 | -7.26 | 14.89 |
| 150 | 4.758 | 0.6898 | 3.403 | 35.93 | 1.493 | -1.42 | 11.60 | -8.08 | 16.14 |
| 155 | 5.431 | 0.7003 | 3.586 | 37.85 | 1.517 | -2.09 | 12.43 | -9.01 | 17.51 |
| 160 | 6.178 | 0.7111 | 3.779 | 39.94 | 1.539 | -2.78 | 13.34 | -10.06 | 19.02 |
| 165 | 7.004 | 0.7222 | 3.983 | 42.21 | 1.559 | -3.49 | 14.32 | -11.25 | 20.69 |
| 170 | 7.916 | 0.7336 | 4.199 | 44.69 | 1.576 | -4.23 | 15.39 | -12.60 | 22.53 |
| 175 | 8.920 | 0.7454 | 4.427 | 47.41 | 1.591 | -4.99 | 16.56 | -14.13 | 24.58 |
| 180 | 10.021 | 0.7575 | 4.669 | 50.40 | 1.603 | -5.76 | 17.82 | -15.88 | 26.86 |
| 185 | 11.226 | 0.7700 | 4.926 | 53.71 | 1.613 | -6.56 | 19.21 | -17.89 | 29.40 |
| 190 | 12.543 | 0.7829 | 5.199 | 57.37 | 1.619 | -7.36 | 20.72 | -20.18 | 32.24 |
| 195 | 13.978 | 0.7962 | 5.491 | 61.45 | 1.622 | -8.18 | 22.38 | -22.82 | 35.43 |
| 200 | 15.537 | 0.8099 | 5.804 | 66.01 | 1.622 | -9.02 | 24.21 | -25.86 | 39.02 |
| 210 | 19.061 | 0.8387 | 6.501 | 76.87 | 1.612 | -10.70 | 28.42 | -33.47 | 47.65 |
| 220 | 23.177 | 0.8697 | 7.314 | 90.70 | 1.587 | -12.41 | 33.56 | -43.79 | 58.82 |
| 225 | 25.476 | 0.8860 | 7.773 | 99.05 | 1.568 | -13.26 | 36.56 | -50.34 | 65.64 |
| 230 | 27.947 | 0.9030 | 8.273 | 108.57 | 1.546 | -14.11 | 39.90 | -58.06 | 73.49 |
| 240 | 33.440 | 0.9391 | 9.421 | 132.00 | 1.487 | -15.79 | 47.81 | -78.13 | 93.11 |
| 250 | 39.728 | 0.9785 | 10.814 | 163.21 | 1.408 | -17.44 | 57.81 | -106.98 | 119.90 |
| 260 | 46.883 | 1.0218 | 12.533 | 205.53 | 1.309 | -19.03 | 70.65 | -149.54 | 157.34 |
| 270 | 54.984 | 1.0699 | 14.690 | 264.21 | 1.186 | -20.57 | 87.45 | -214.19 | 211.12 |
| 275 | 59.415 | 1.0960 | 15.980 | 302.15 | 1.115 | -21.32 | 97.83 | -259.08 | 246.93 |
| 280 | 64.113 | 1.1238 | 17.447 | 347.77 | 1.036 | -22.05 | 109.89 | -315.93 | 290.94 |
| 290 | 74.356 | 1.1850 | 21.053 | 470.95 | 0.853 | -23.47 | 140.65 | -483.00 | 414.21 |
| 300 | 85.805 | 1.2555 | 25.893 | 660.79 | 0.632 | -24.85 | 184.17 | -772.23 | 614.27 |
| 325 | 120.387 | 1.4943 | 49.072 | 1964.42 | -0.178 | -28.50 | 419.46 | -3353.74 | 2160.46 |
| 350 | 165.125 | 1.9252 | 135.176 | 13018.24 | -1.801 | -35.41 | 1504.78 | -36221.30 | 17927.28 |

[a] °C. [b] bar. [c] $kg^{1/2}\ mole^{-1/2}$. [d] $kcal\ kg^{1/2}\ mole^{-3/2}$. [e] $cal\ kg^{1/2}\ mole^{-3/2}(°K)^{-1}$.
[f] $cm^3\ kg^{1/2}\ mole^{-3/2}$. [g] $cm^3\ kg^{1/2}\ mole^{-3/2}\ bar^{-1}$. [h] $cm^3\ kg^{1/2}\ mole^{-3/2}\ (°K)^{-1}$.

TABLE 18

Summary of molal Debye-Hückel B parameters for the steam-saturated liquid phase

| t[a] (°C) | P[b] (bar) | $B_Y \times 10^{-7}$[c] | $B_H \times 10^{-9}$[d] | $B_J \times 10^{-8}$[e] | $B_E \times 10^{-9}$[d] | $B_{CV} \times 10^{-7}$[e] | $B_V \times 10^{-8}$[f] | $B_K \times 10^{-4}$[g] | $B_{Ex} \times 10^{-5}$[h] |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.006 | 0.3247 | 9.04 | 1.25 | 9.00 | 1.40 | 5.94 | -0.36 | -3.36 |
| 5 | 0.009 | 0.3254 | 9.68 | 1.32 | 9.69 | 1.38 | 4.28 | -0.32 | -3.30 |
| 10 | 0.012 | 0.3261 | 10.36 | 1.39 | 10.39 | 1.37 | 2.65 | -0.26 | -3.22 |
| 15 | 0.017 | 0.3268 | 11.07 | 1.46 | 11.09 | 1.35 | 1.05 | -0.19 | -3.15 |
| 20 | 0.023 | 0.3275 | 11.82 | 1.53 | 11.80 | 1.33 | -0.51 | -0.12 | -3.12 |
| 25 | 0.032 | 0.3283 | 12.60 | 1.59 | 12.51 | 1.30 | -2.07 | -0.05 | -3.12 |
| 30 | 0.042 | 0.3291 | 13.41 | 1.65 | 13.23 | 1.27 | -3.64 | 0.02 | -3.16 |
| 35 | 0.056 | 0.3299 | 14.24 | 1.70 | 13.94 | 1.24 | -5.24 | 0.09 | -3.23 |
| 40 | 0.074 | 0.3307 | 15.11 | 1.74 | 14.65 | 1.21 | -6.87 | 0.16 | -3.32 |
| 45 | 0.096 | 0.3316 | 15.99 | 1.79 | 15.36 | 1.17 | -8.56 | 0.23 | -3.44 |
| 50 | 0.123 | 0.3325 | 16.89 | 1.82 | 16.06 | 1.13 | -10.32 | 0.31 | -3.58 |
| 55 | 0.158 | 0.3334 | 17.81 | 1.86 | 16.75 | 1.08 | -12.15 | 0.39 | -3.75 |
| 60 | 0.199 | 0.3343 | 18.75 | 1.89 | 17.43 | 1.03 | -14.07 | 0.48 | -3.94 |
| 65 | 0.250 | 0.3352 | 19.70 | 1.92 | 18.09 | 0.97 | -16.09 | 0.57 | -4.15 |
| 70 | 0.312 | 0.3362 | 20.67 | 1.95 | 18.74 | 0.92 | -18.22 | 0.67 | -4.38 |
| 75 | 0.386 | 0.3371 | 21.65 | 1.98 | 19.37 | 0.85 | -20.47 | 0.78 | -4.63 |
| 80 | 0.474 | 0.3381 | 22.65 | 2.01 | 19.98 | 0.79 | -22.85 | 0.91 | -4.90 |
| 85 | 0.578 | 0.3391 | 23.66 | 2.04 | 20.57 | 0.72 | -25.38 | 1.04 | -5.20 |
| 90 | 0.701 | 0.3401 | 24.68 | 2.06 | 21.14 | 0.64 | -28.05 | 1.19 | -5.52 |
| 95 | 0.845 | 0.3411 | 25.72 | 2.09 | 21.68 | 0.57 | -30.90 | 1.36 | -5.87 |
| 100 | 1.013 | 0.3422 | 26.77 | 2.12 | 22.20 | 0.49 | -33.92 | 1.54 | -6.24 |
| 105 | 1.208 | 0.3432 | 27.84 | 2.15 | 22.69 | 0.40 | -37.14 | 1.75 | -6.64 |
| 110 | 1.433 | 0.3443 | 28.93 | 2.19 | 23.14 | 0.32 | -40.56 | 1.97 | -7.07 |
| 115 | 1.691 | 0.3454 | 30.03 | 2.22 | 23.57 | 0.23 | -44.21 | 2.23 | -7.54 |
| 120 | 1.985 | 0.3465 | 31.14 | 2.26 | 23.96 | 0.13 | -48.10 | 2.51 | -8.04 |
| 125 | 2.321 | 0.3476 | 32.28 | 2.29 | 24.31 | 0.04 | -52.24 | 2.82 | -8.58 |
| 130 | 2.701 | 0.3487 | 33.43 | 2.33 | 24.63 | -0.06 | -56.66 | 3.17 | -9.16 |
| 135 | 3.130 | 0.3498 | 34.61 | 2.38 | 24.90 | -0.16 | -61.39 | 3.57 | -9.79 |
| 140 | 3.613 | 0.3510 | 35.80 | 2.42 | 25.14 | -0.26 | -66.43 | 4.00 | -10.47 |
| 145 | 4.154 | 0.3521 | 37.02 | 2.48 | 25.34 | -0.36 | -71.83 | 4.49 | -11.21 |
| 150 | 4.758 | 0.3533 | 38.27 | 2.53 | 25.49 | -0.47 | -77.61 | 5.04 | -12.01 |
| 155 | 5.431 | 0.3545 | 39.54 | 2.59 | 25.60 | -0.57 | -83.79 | 5.65 | -12.88 |
| 160 | 6.178 | 0.3556 | 40.85 | 2.66 | 25.66 | -0.68 | -90.42 | 6.33 | -13.83 |
| 165 | 7.004 | 0.3568 | 42.19 | 2.74 | 25.68 | -0.79 | -97.53 | 7.11 | -14.86 |
| 170 | 7.916 | 0.3580 | 43.57 | 2.82 | 25.65 | -0.89 | -105.17 | 7.97 | -15.99 |
| 175 | 8.920 | 0.3592 | 44.99 | 2.92 | 25.57 | -1.00 | -113.39 | 8.95 | -17.23 |
| 180 | 10.021 | 0.3605 | 46.46 | 3.03 | 25.44 | -1.11 | -122.23 | 10.06 | -18.58 |
| 185 | 11.226 | 0.3617 | 47.98 | 3.15 | 25.26 | -1.21 | -131.76 | 11.31 | -20.08 |
| 190 | 12.543 | 0.3629 | 49.56 | 3.29 | 25.03 | -1.31 | -142.05 | 12.73 | -21.73 |
| 195 | 13.978 | 0.3642 | 51.22 | 3.45 | 24.75 | -1.42 | -153.17 | 14.34 | -23.56 |
| 200 | 15.537 | 0.3655 | 52.95 | 3.63 | 24.41 | -1.52 | -165.21 | 16.19 | -25.59 |
| 210 | 19.061 | 0.3681 | 56.70 | 4.07 | 23.59 | -1.71 | -192.46 | 20.70 | -30.38 |
| 220 | 23.177 | 0.3707 | 60.91 | 4.65 | 22.55 | -1.89 | -224.76 | 26.69 | -36.40 |
| 225 | 25.476 | 0.3721 | 63.23 | 5.00 | 21.96 | -1.97 | -243.18 | 30.40 | -39.99 |
| 230 | 27.947 | 0.3734 | 65.72 | 5.40 | 21.31 | -2.05 | -263.37 | 34.72 | -44.07 |
| 240 | 33.440 | 0.3762 | 71.30 | 6.39 | 19.86 | -2.20 | -309.99 | 45.69 | -54.01 |
| 250 | 39.728 | 0.3792 | 77.90 | 7.69 | 18.20 | -2.32 | -366.91 | 60.96 | -67.13 |
| 260 | 46.883 | 0.3822 | 85.80 | 9.43 | 16.33 | -2.43 | -437.32 | 82.69 | -84.81 |
| 270 | 54.984 | 0.3855 | 95.43 | 11.77 | 14.25 | -2.51 | -525.73 | 114.46 | -109.22 |
| 275 | 59.415 | 0.3871 | 101.06 | 13.25 | 13.13 | -2.54 | -578.63 | 135.88 | -124.99 |
| 280 | 64.113 | 0.3889 | 107.34 | 14.99 | 11.95 | -2.57 | -638.73 | 162.40 | -143.94 |
| 290 | 74.356 | 0.3926 | 122.35 | 19.55 | 9.43 | -2.61 | -786.27 | 237.54 | -195.13 |
| 300 | 85.805 | 0.3965 | 141.64 | 26.23 | 6.66 | -2.62 | -984.09 | 361.08 | -274.07 |
| 325 | 120.387 | 0.4085 | 224.59 | 67.34 | -1.63 | -2.60 | -1926.64 | 1330.59 | -814.22 |
| 350 | 165.125 | 0.4256 | 472.54 | 353.14 | -13.27 | -2.63 | -5336.98 | 10974.44 | -5197.56 |

[a] °C. [b] bar. [c] $kg^{1/2}\ cm^{-1}\ mole^{-3/2}$. [d] $cal\ kg^{1/2}\ cm^{-1}\ mole^{-3/2}$. [e] $cal\ kg^{1/2}\ cm^{-1}\ mole^{-3/2}\ (°K)^{-1}$. [f] $cm^2\ kg^{1/2}\ mole^{-3/2}$. [g] $cm^2\ kg^{1/2}\ mole^{-3/2}\ bar^{-1}$. [h] $cm^2\ kg^{1/2}\ mole^{-3/2}\ (°K)^{-1}$

pressure (fig. 50), and the isopleths for $A_{Ex}$ in figure 53 wrap around the critical point. In contrast, $B_{Ex}$ as an isobaric function of temperature exhibits extrema at pressures $\gtrsim$ 4 kb as well as minima at higher temperatures (fig. 52). $B_{Ex}$ is negative, except at high pressures > 4 kb from ~ 300° to ~ 500°C, where it maximizes as a function of temperature at constant pressure. Note also in figure 52 that $B_{Ex}$ maximizes with decreasing temperature at constant pressure below a kilobar in the vicinity of 35°C in response to the influence of $(\partial \ln \varepsilon/\partial P)_T$ and $\beta$, which exhibit the same behavior. The high pressure extrema in figure 52 are manifested by the crossing isotherms in figure 51 and the high pressure minima in the isopleths shown in figure 54.

## SUMMARY OF PARAMETERS FOR SATURATED LIQUID $H_2O$

Computed values of $A_\gamma$, $B_\gamma$, $A_H$, $B_H$, $A_J$, $B_J$, $A_E$, $B_E$, $A_{C_V}$, $B_{C_V}$, $A_V$, $B_V$, $A_\kappa$, $B_\kappa$, $A_{Ex}$, and $B_{Ex}$ in liquid $H_2O$ along the saturation curve are given in tables 17 and 18 at closely spaced intervals from 0° to 350°C to facilitate analysis of experimental data.

## CONCLUDING REMARKS

Owing to the much greater increase in $A_\gamma$ relative to the increase in $B_\gamma$ with increasing temperature, activity coefficients computed from the Debye-Hückel equation (for a constant å) decrease with increasing temperature at constant ionic strength. At high temperatures and low pressures the decrease becomes dramatic as temperature increases and/or

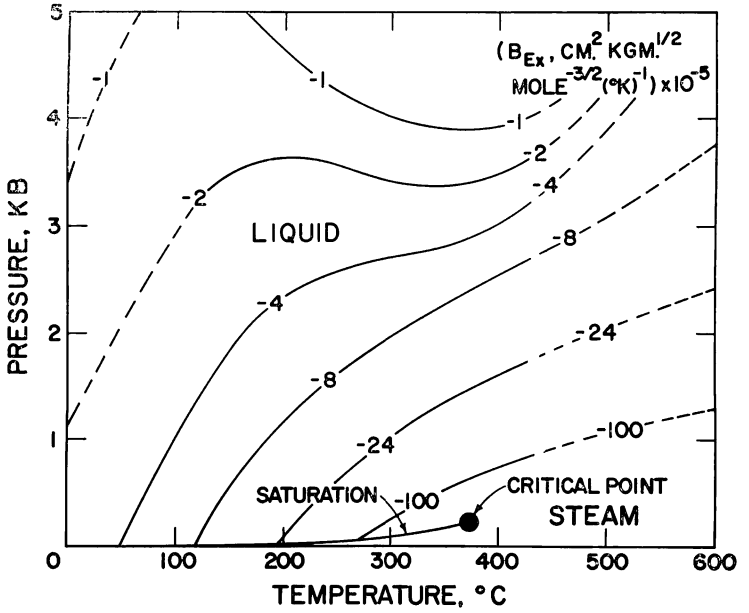


Fig. 54. Isopleths of $B_{Ex}$ (labeled in $(cm^2 kg^{1/2} mole^{-3/2}(°K)^{-1}) \times 10^{-5}$) as a function of pressure and temperature (table 16 and figs. 51 and 52).

pressure decreases, which favors increasing mineral solubilities and decreasing degrees of formation of aqueous complexes. However, dissociation constants and $b_\gamma$ become small at high temperatures (Quist and Marshall, 1966, 1968a, b, c, and d, 1969; Helgeson and James, 1968; Helgeson, 1967a, 1969) where true ionic strengths of 0.01 or less occur in concentrated electrolyte solutions owing to the high degrees of formation achieved by neutral complexes in a medium of relatively low dielectric constant. As a consequence, Debye-Hückel theory becomes increasingly applicable to solutions with progressively higher concentrations as temperature increases and/or pressure decreases. The increasing electrostatic contribution to ion association thus overcomes the effect of decreasing ionic activity coefficients on the formation of aqueous complexes with increasing temperature and/or decreasing pressure.

The tables and diagrams presented in the foregoing pages were generated with the aid of a comprehensive computer program which can be used to calculate all the high-temperature/pressure thermodynamic/electrostatic properties of $H_2O$ (Helgeson and Kirkham, 1974a). Hopefully, such calculations will stimulate more experimental research on the thermodynamic properties of aqueous electrolytes at high pressures and temperatures. Precise density measurements under these conditions are needed to test and refine equations of state for electrolytes (Helgeson and Kirkham, 1975a). With the aid of the $A_V$ and $B_V$ values given above, standard state data can be extracted from such measurements to calculate the thermodynamic properties of aqueous species in hydrothermal solutions at high pressures and temperatures (Helgeson and Kirkham, 1975b).

ACKNOWLEDGMENTS

The work reported here was supported in part by NSF Grants GA 25314, GA 36023, GA 35888, and the donors of the Petroleum Research Fund, administered by the American Chemical Society, under PRF #5356-AC2. Additional funds provided by the Kennecott Copper Corporation, the Anaconda Company, and the Committee on Research at the University of California, Berkeley, are also acknowledged with thanks. We are indebted to Philip Lehot, David Bice, Yoshi Hidaka, David Pitou, Joan Delany, and Joachim Hampel for assistance with programming, plotting, drafting, and preparation of the manuscript. Our gratitude also to A. J. Ellis, J. W. Cobble, and C. W. Burnham for critical reviews of the paper, and to Debbie Aoki for typing and retyping the manuscript.

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